





Precession Diffraction: The  
Philosopher's Stone of Electron  
Crystallography?

- 
- Many methods exist for obtaining diffraction information
    - Selected Area
    - Nanodiffraction and variants
    - CBED
  - All are complicated to interpret
  - Reciprocal space is right, but intensities depend upon thickness, tilt etc

# What PED can do

- 
- We would like a method where not just the positions of the spots, but also the intensities could be used.
  - Not rigorously equivalent to simple kinematical diffraction, but has many similarities
    - If the structure factor is large  $\rightarrow$  Intensity is large
    - Useful for fingerprinting structures
    - Often does not need calculations to interpret

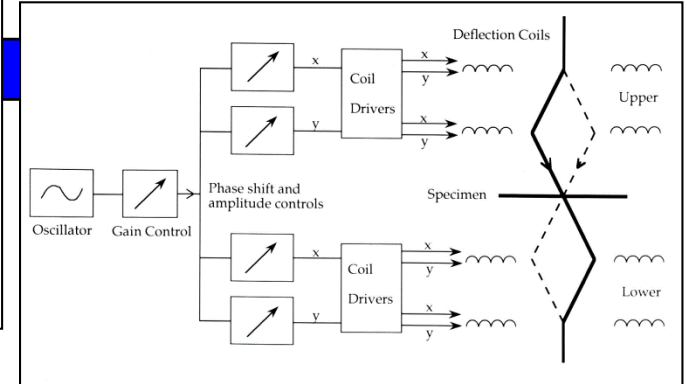
# History – Electron Precession (1993)

## Double conical beam-rocking system for measurement of integrated electron diffraction intensities

R. Vincent, P.A. Midgley

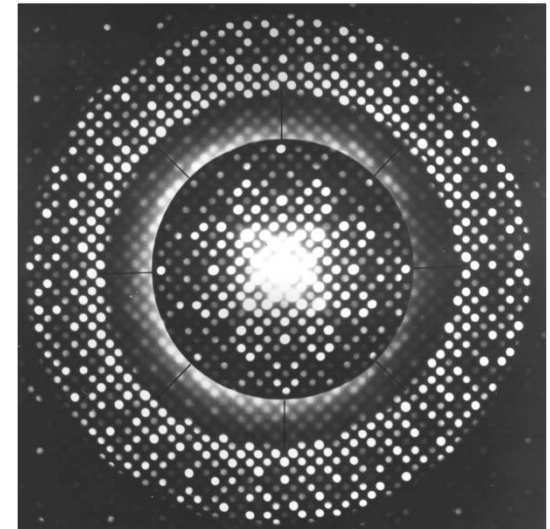
*H.H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol BS8 1TL, UK*

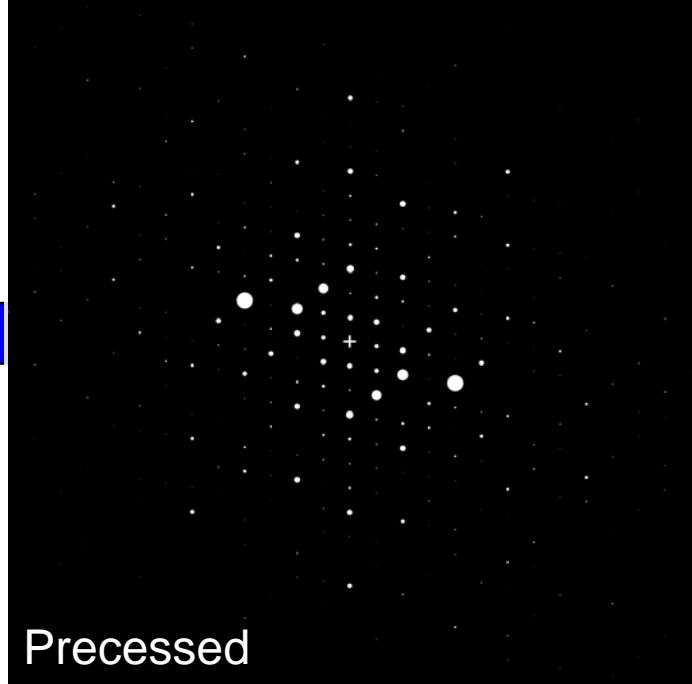
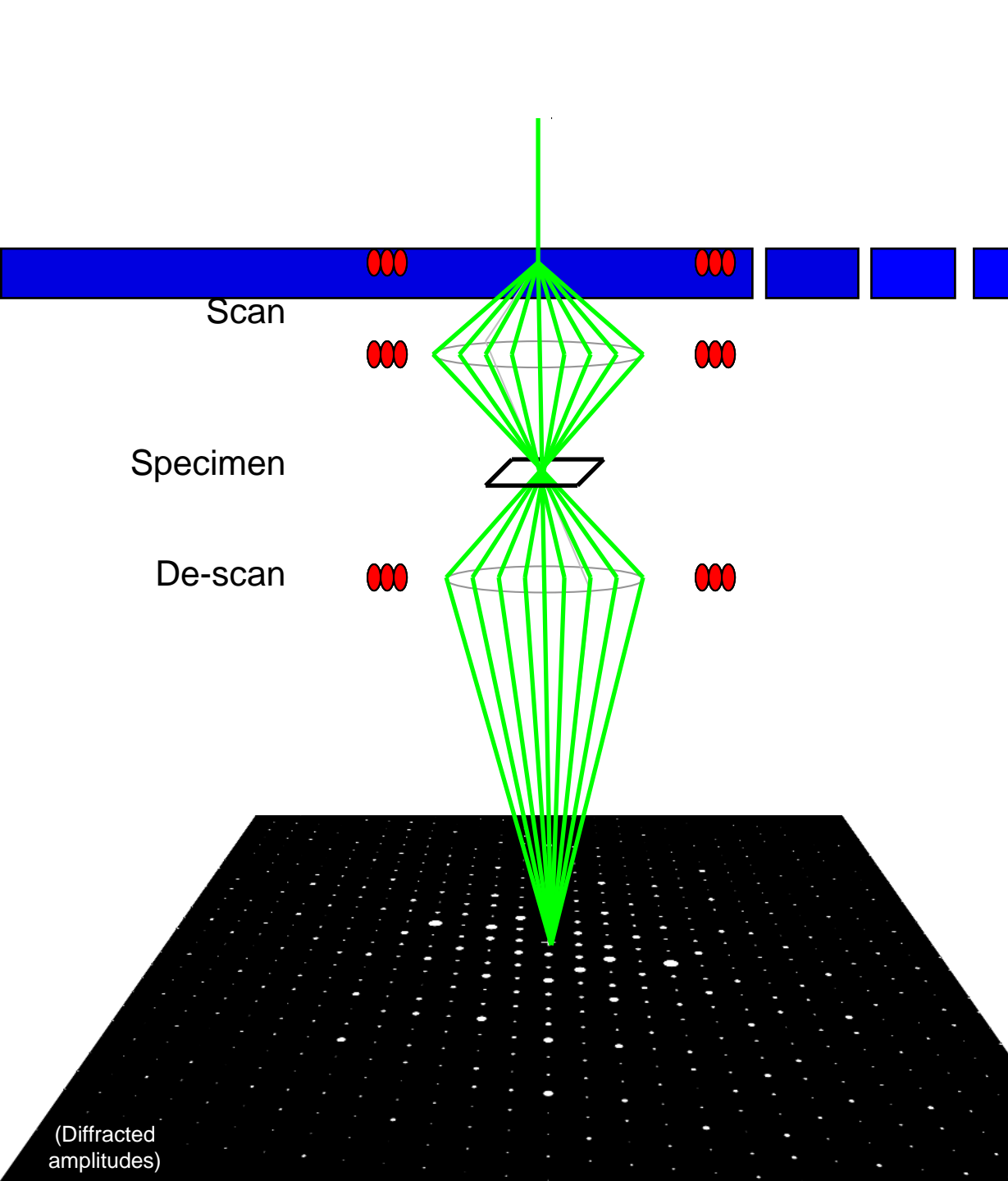
(Received 26 July 1993; in final form 4 October 1993)



## Advantages:

1. PRECESSION → MANY MORE REFLECTIONS INTERCEPTED BY EWALD SPHERE → LARGE DATA SET
2. DIFFRACTED INTENSITIES DETERMINED BY INTEGRATING THROUGH BRAGG CONDITION → NO BRANCH STRUCTURE  
 $\therefore I_g \rightarrow |U_g|^2$  (NOT PARTIAL S.F.)
3. REDUCES NON-SYSTEMATIC DYNAMICAL EFFECTS
4. FOCUSED PROBE → HIGH SPATIAL RESOLUTION ( $\sim 0.1\mu\text{m}$ )



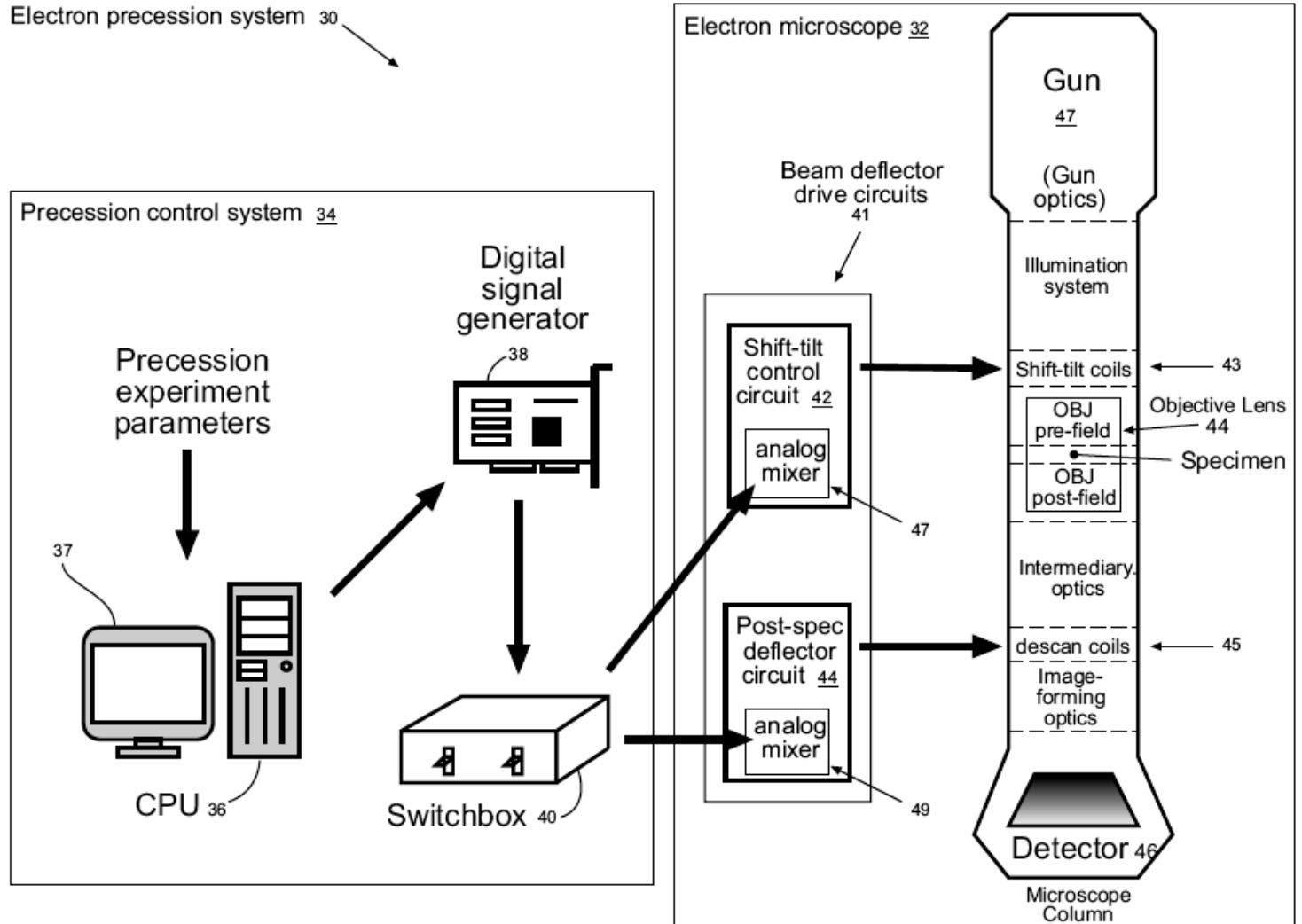
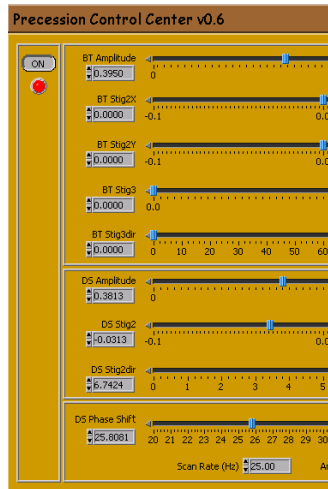


(Ga,In)<sub>2</sub>SnO<sub>5</sub> Intensities  
412Å crystal thickness

Precessional  
Precession  
Diffraction Pattern

# Precession System

US patent application:  
 "A hollow-cone electron diffraction system".  
 Application serial number 60/531,641, Dec 2004.



# SPINNING STAR : UNIVERSAL INTERFASE FOR PRECESSION ELECTRON DIFFRACTION FOR ANY TEM ( 120 -200 -300 KV )

- Can be easily retrofitted to any TEM 100- 300 KV
- precession is possible for any beam size 300 - 50 nm
- Precession is possible for a parallel or convergent beam
- precession eliminates false spots to ED pattern that belong to dynamical contributions
- precession angle can vary continuously ( $0^{\circ}$ - $3^{\circ}$ ) to observe true crystallographic symmetry variation
- Software ELD for easy quantification of ED intensities and automatic symmetry ( point, space group ) research




- Easily interfaced to electron diffractometer for automatic 3D structure determination



**NanoMEGAS**

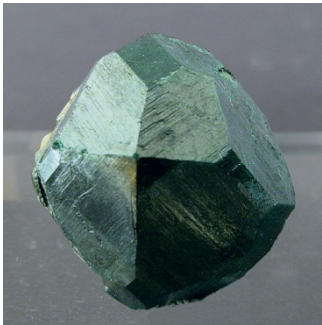
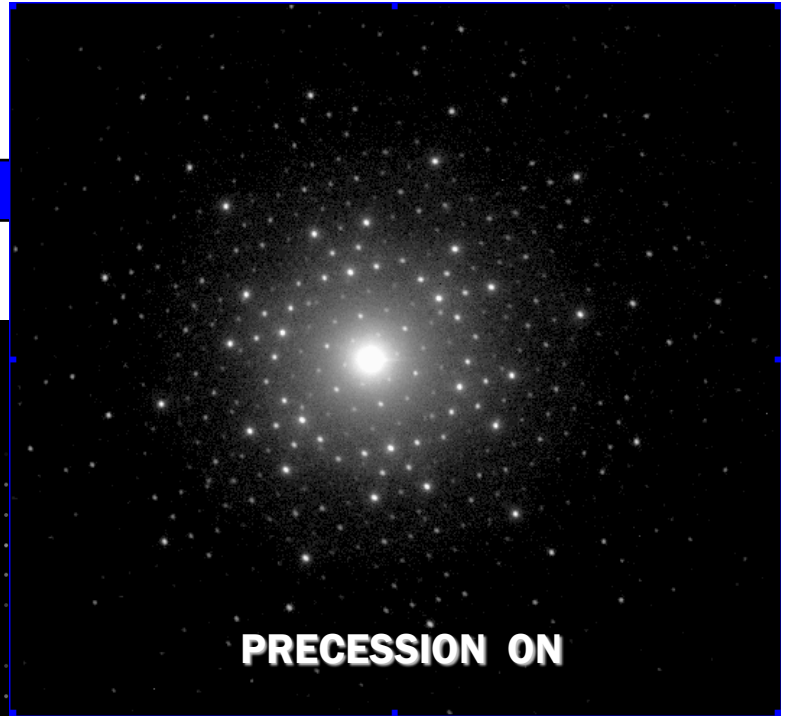
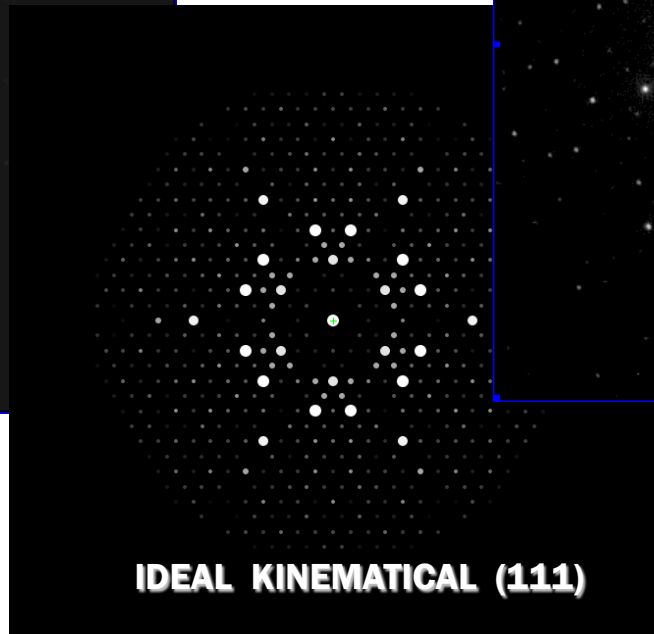
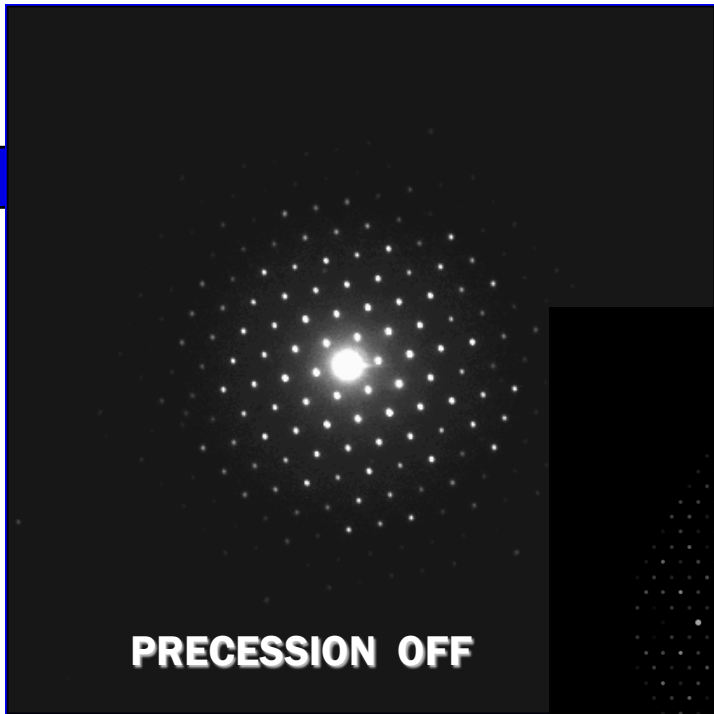
Advanced Tools for electron diffraction

# Examples:

- 
- Complicated Structures
    - Hard to interpret SAED
    - Simple to interpret PED
  - EDS
    - Elemental ratio's depend upon orientation in standard mode
    - Weak to no dependence with PED



# APPLICATION : FIND TRUE CRYSTAL SYMMETRY



**UVAROVITE (111)**

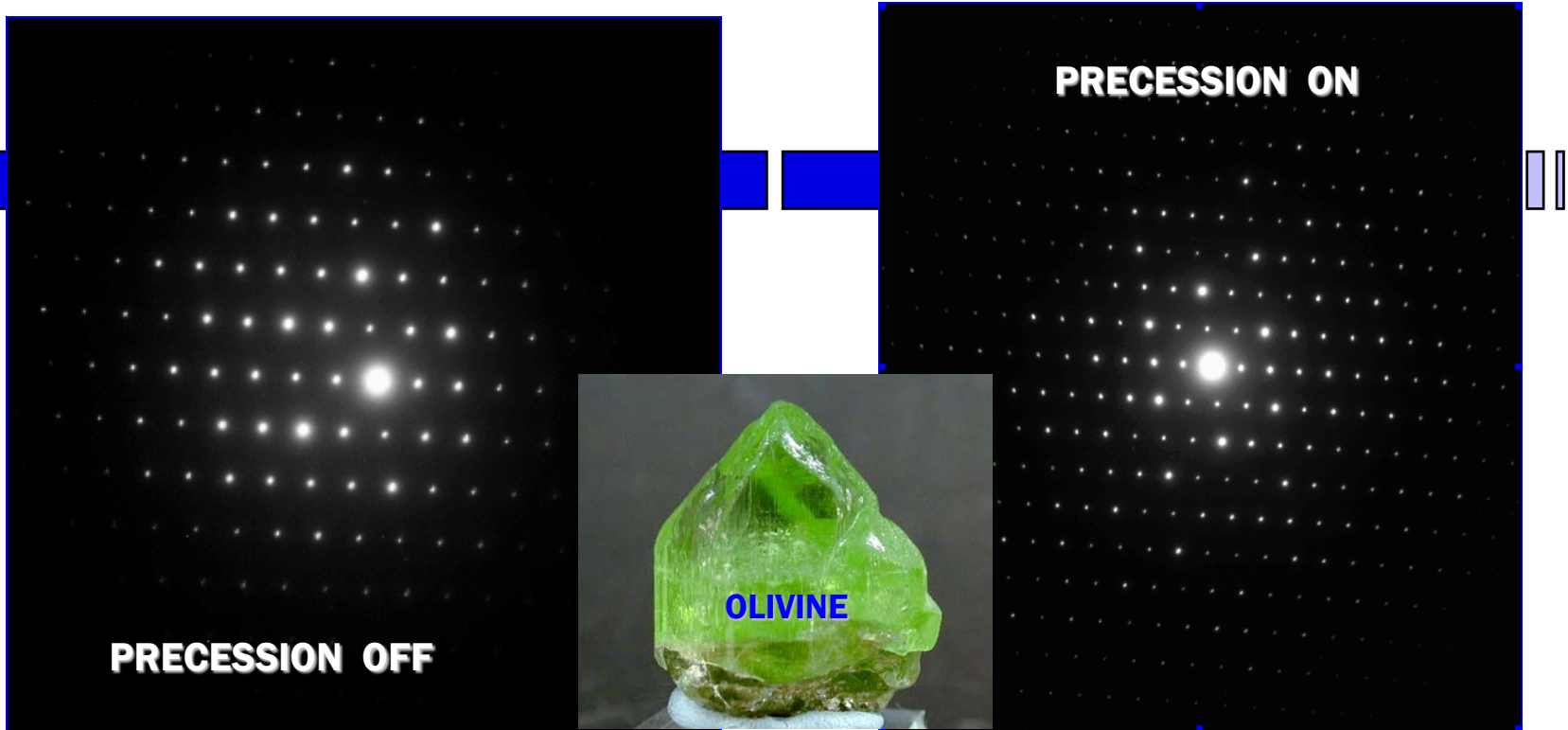
Courtesy M.Gemmi  
Univ of Milano



**NanoMEGAS**  
Advanced Tools for electron diffraction



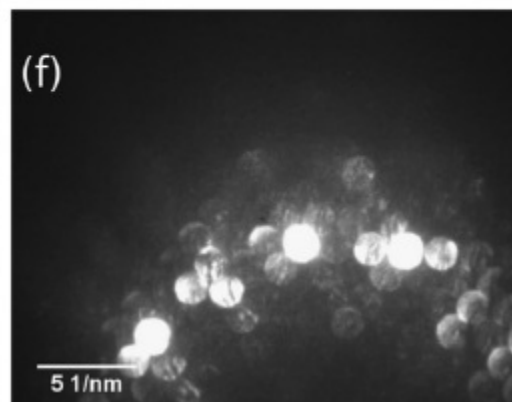
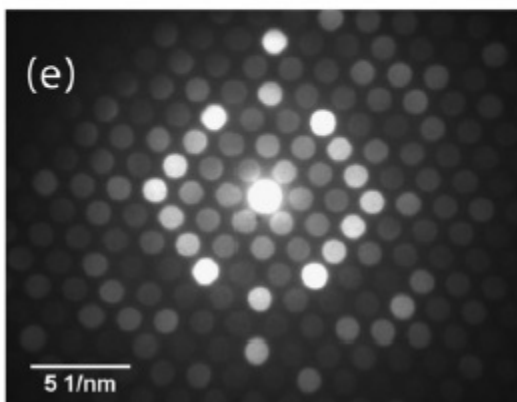
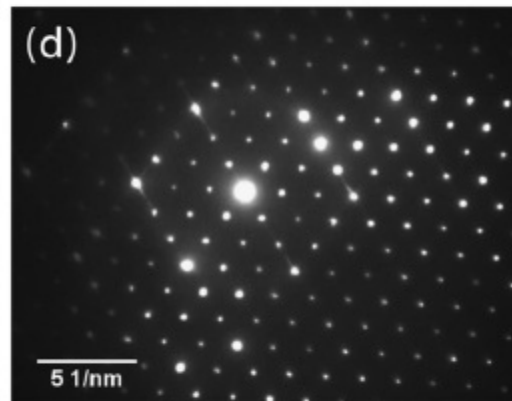
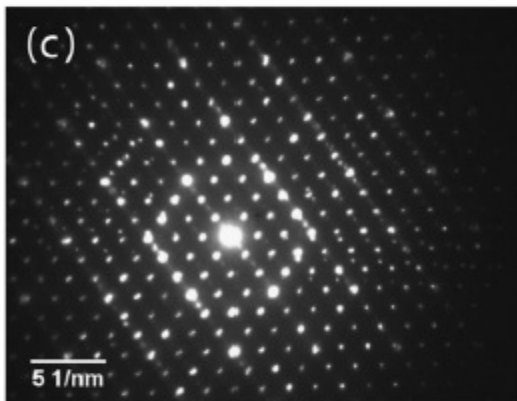
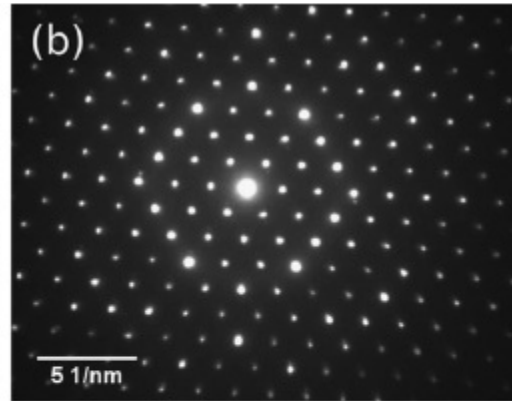
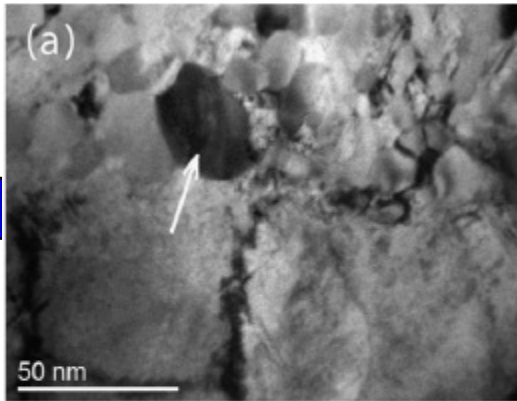
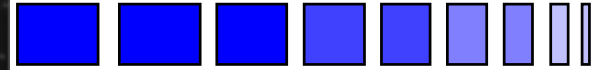
# APPLICATION : PERFECT CRYSTAL ORIENTATION



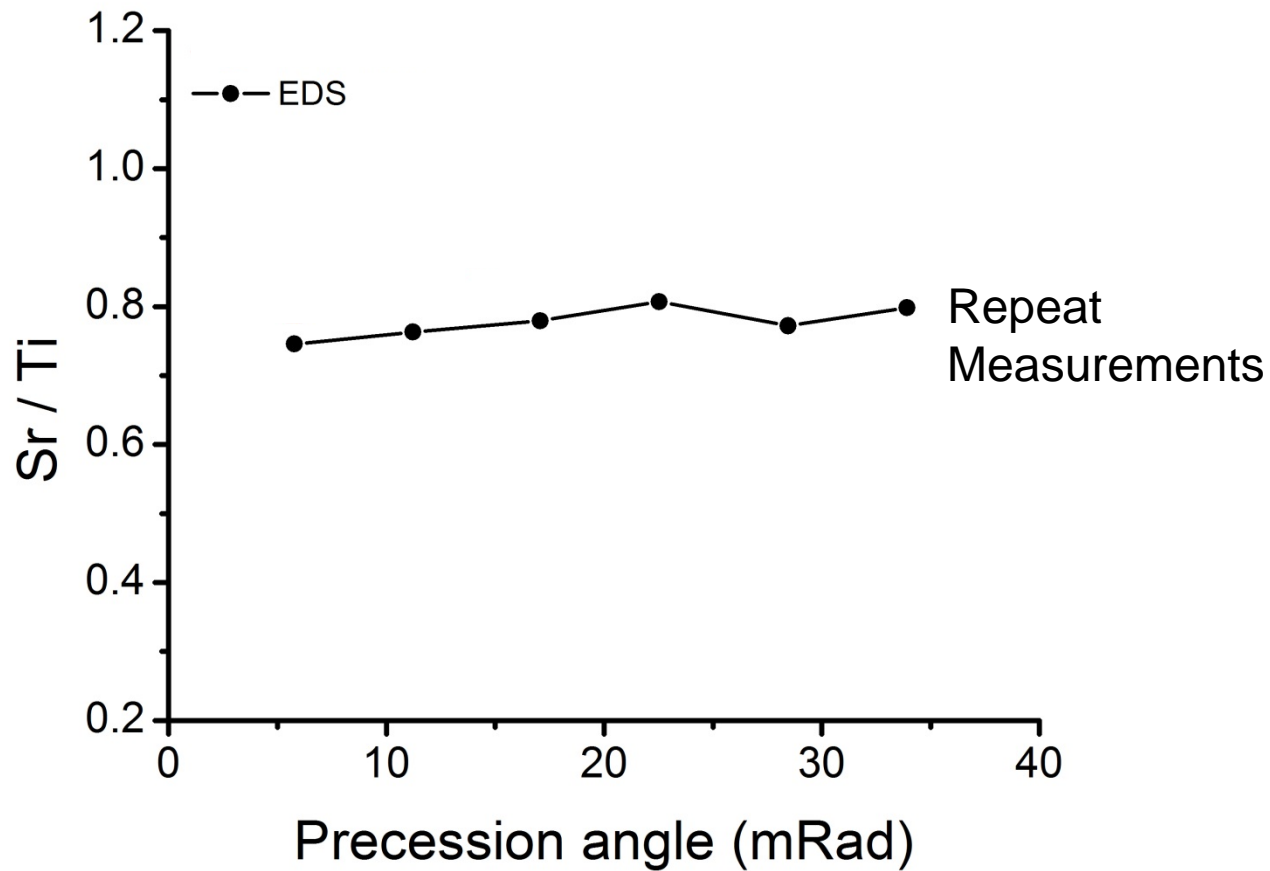
**Crystals –specially minerals -usually grow in platelet or fiber shape and results difficult to orient perfectly in a particular zone axis; in this example olivine crystals are perfectly oriented after precession is on.**




# Carbide



# EDS, on zone ( $\text{SrTiO}_3$ )

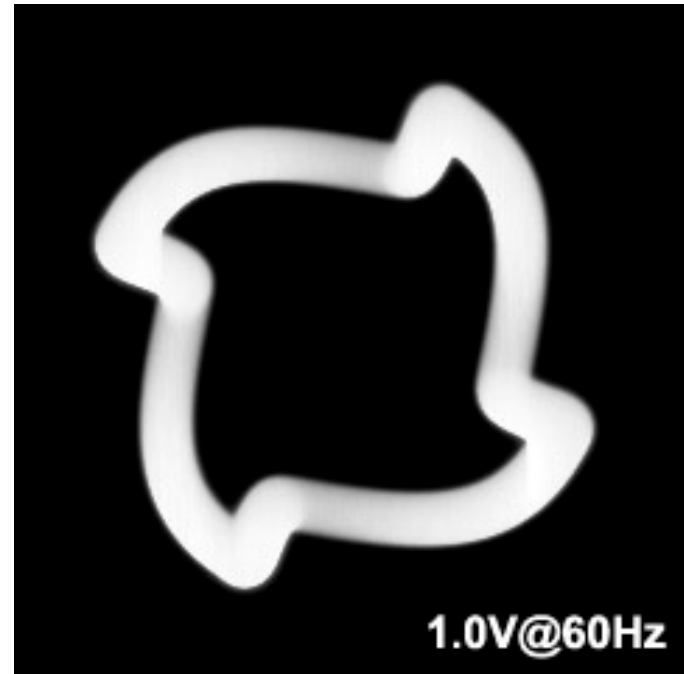
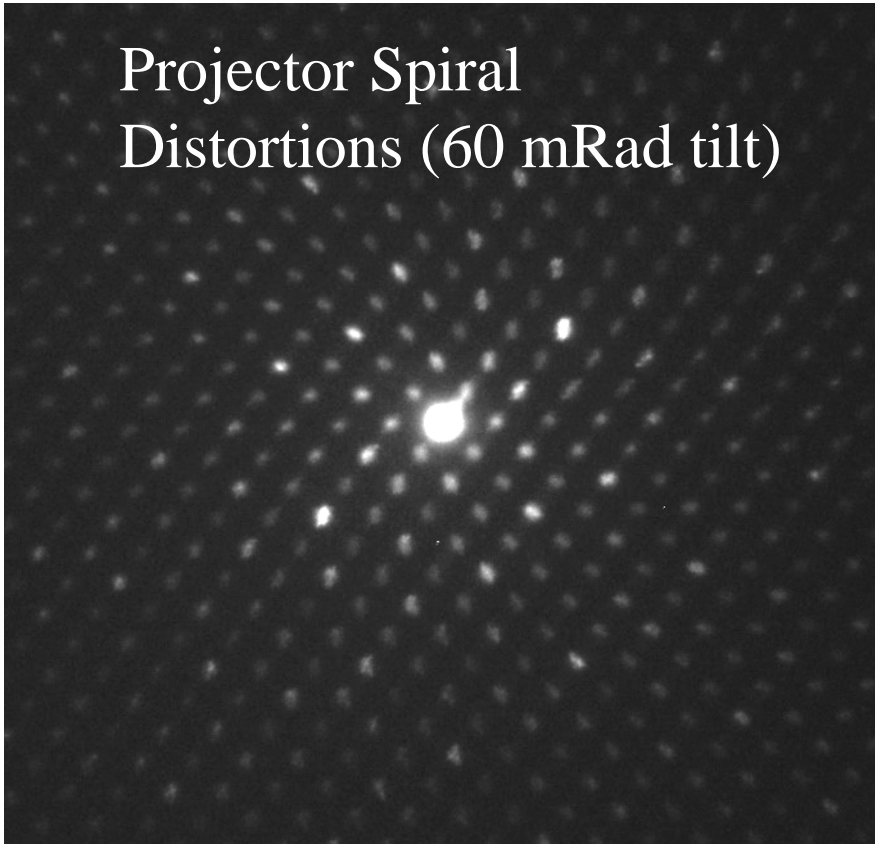


# Practical Use

- 
- Two commercial systems (one hardware, another software) are available
  - Not complicated, and could probably be written in scripting language
  - Alignment can be tricky – it always is
  - Not rocket science to use

# Some Practical Issues

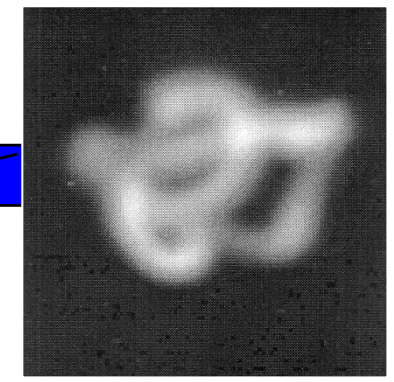
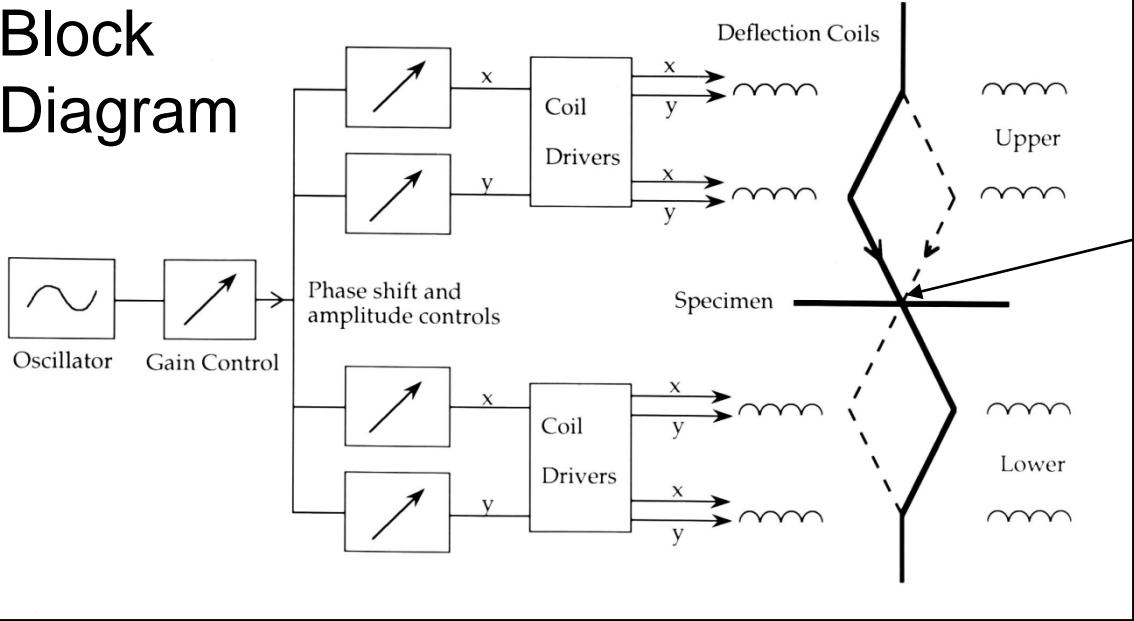
Projector Spiral  
Distortions (60 mRad tilt)



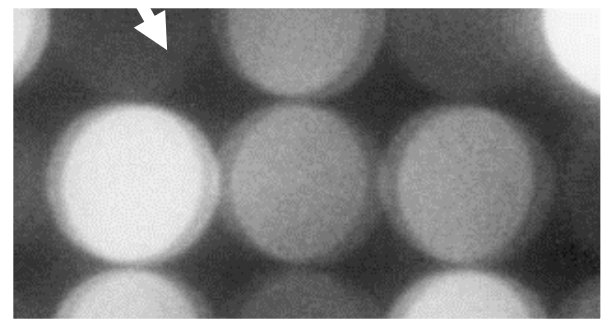
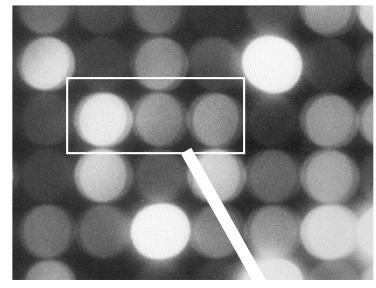
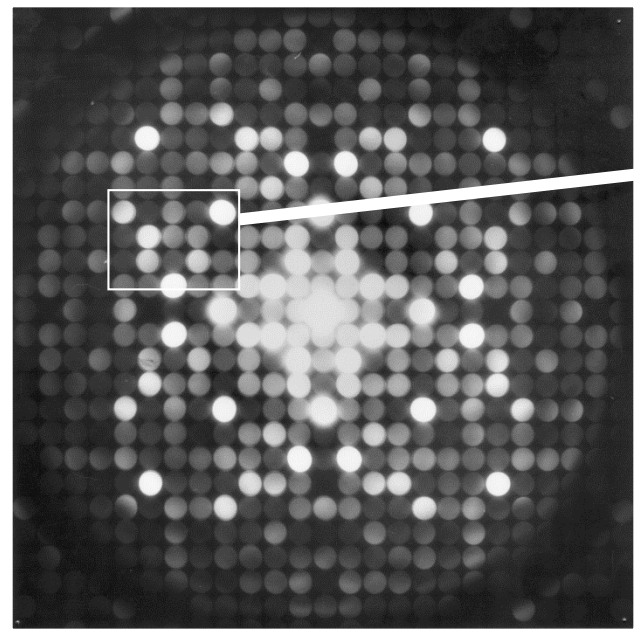
Bi-polar push-pull circuit  
(H9000)



# Block Diagram



‘Aberrations’

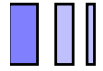


# Optics

Idealized Diagram

Better Diagram

Postfield Misaligned



Scan



Objective  
Prefield



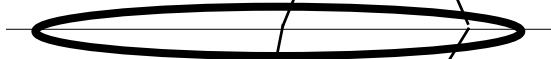
Sample



Objective  
Postfield



Descan



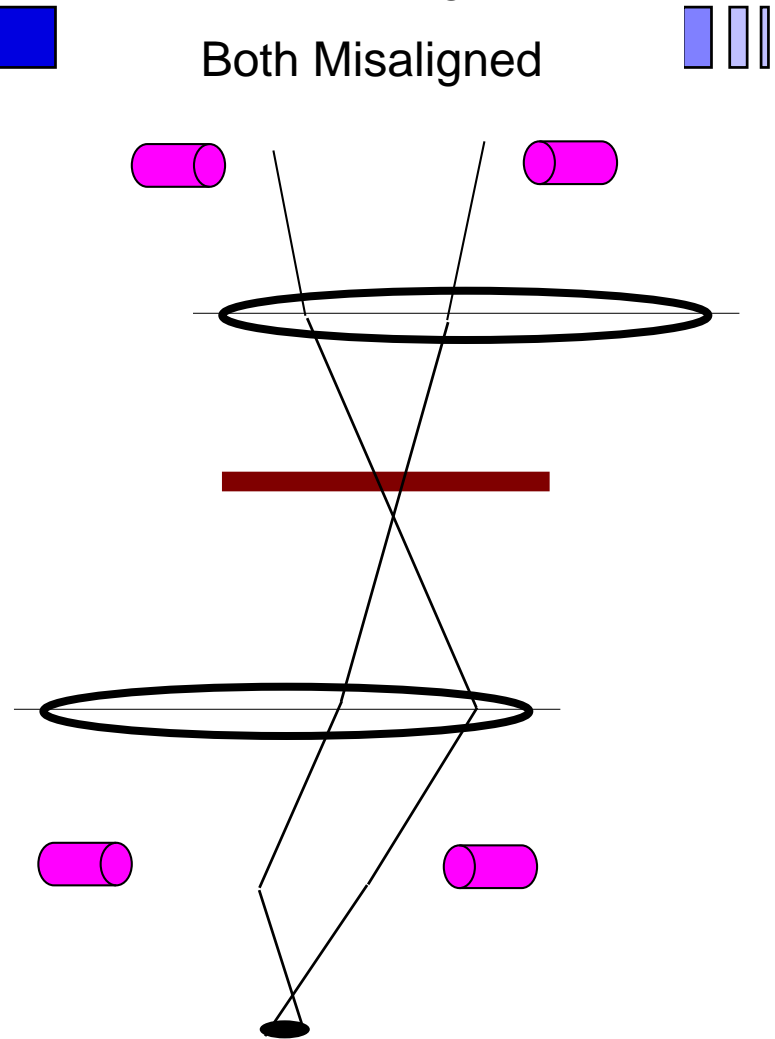
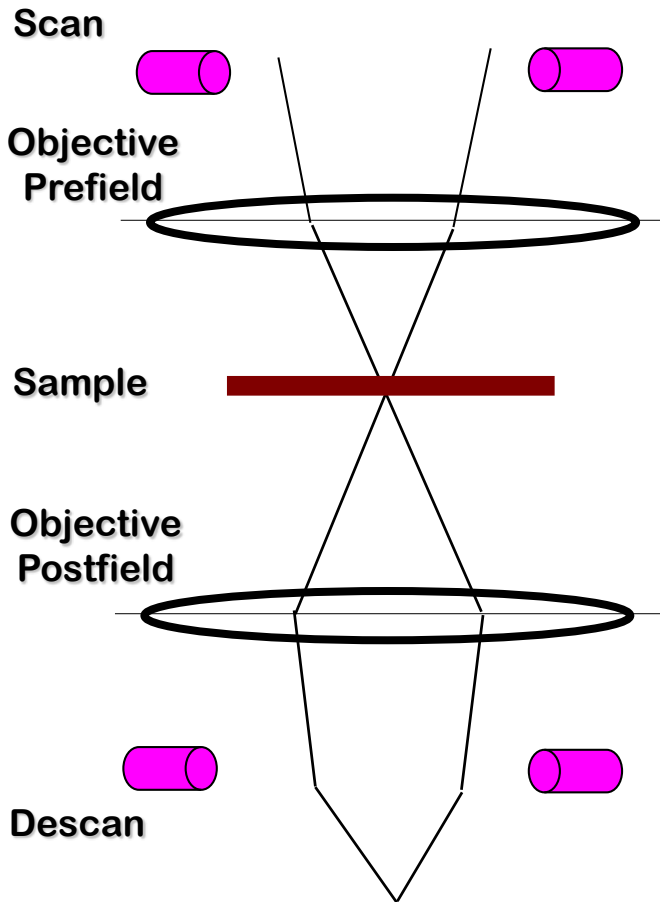


# Optics

Idealized Diagram

Correct Diagram

Both Misaligned



# One Consequence

- Prefield/Postfield displacements of beam

$$d^{\text{Pre}} = 1/(2\pi) \nabla \chi^{\text{Pre}}(s-t^{\text{Pre}}); \quad s = \text{Scan}$$

$$d^{\text{Post}} = 1/(2\pi) \nabla \chi^{\text{Post}}(s-t^{\text{Post}}) - s^{\text{D}}\theta \quad s^{\text{D}} = \text{DeScan}$$

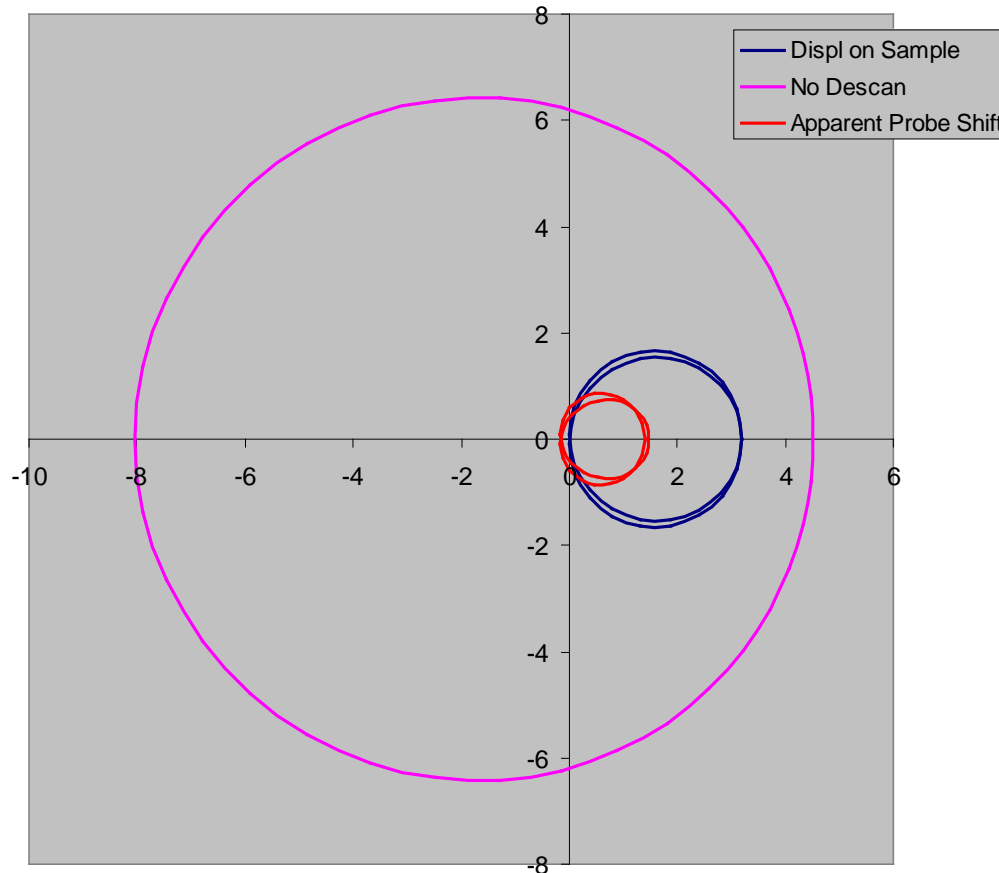
$$\nabla \chi(u)/(2\pi) = \Delta z\theta + C_s\theta^3$$

Total apparent displacement is the sum

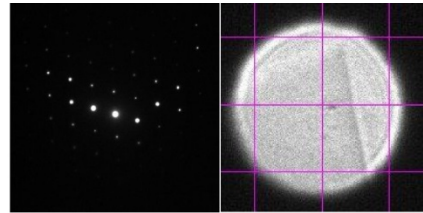
$$\begin{aligned} d^{\text{Nett}} &= d^{\text{Pre}} + d^{\text{Post}} \\ &= \Delta z^{\text{Pre}}(\theta - \theta^{\text{Pre}}) + \Delta z^{\text{Post}}(\theta - \theta^{\text{Post}}) \\ &\quad + C_s^{\text{Pre}}(\theta - \theta^{\text{Pre}})^3 + C_s^{\text{Post}}(\theta - \theta^{\text{Post}})^3 \\ &\quad - s^{\text{D}}\theta \end{aligned}$$

# Probe and Displacements (nm)

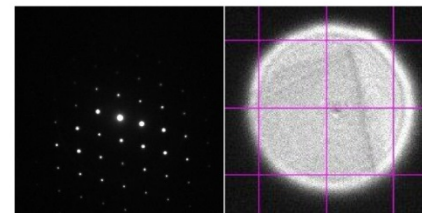
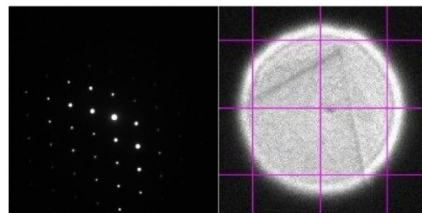
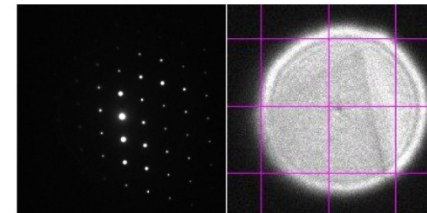
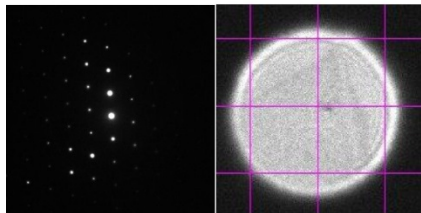
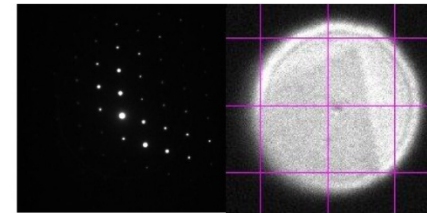
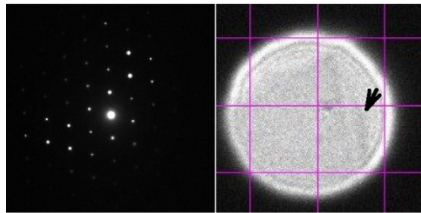
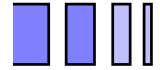
Prefield misalignment 1 mRad; Postfield -1.0 mRad



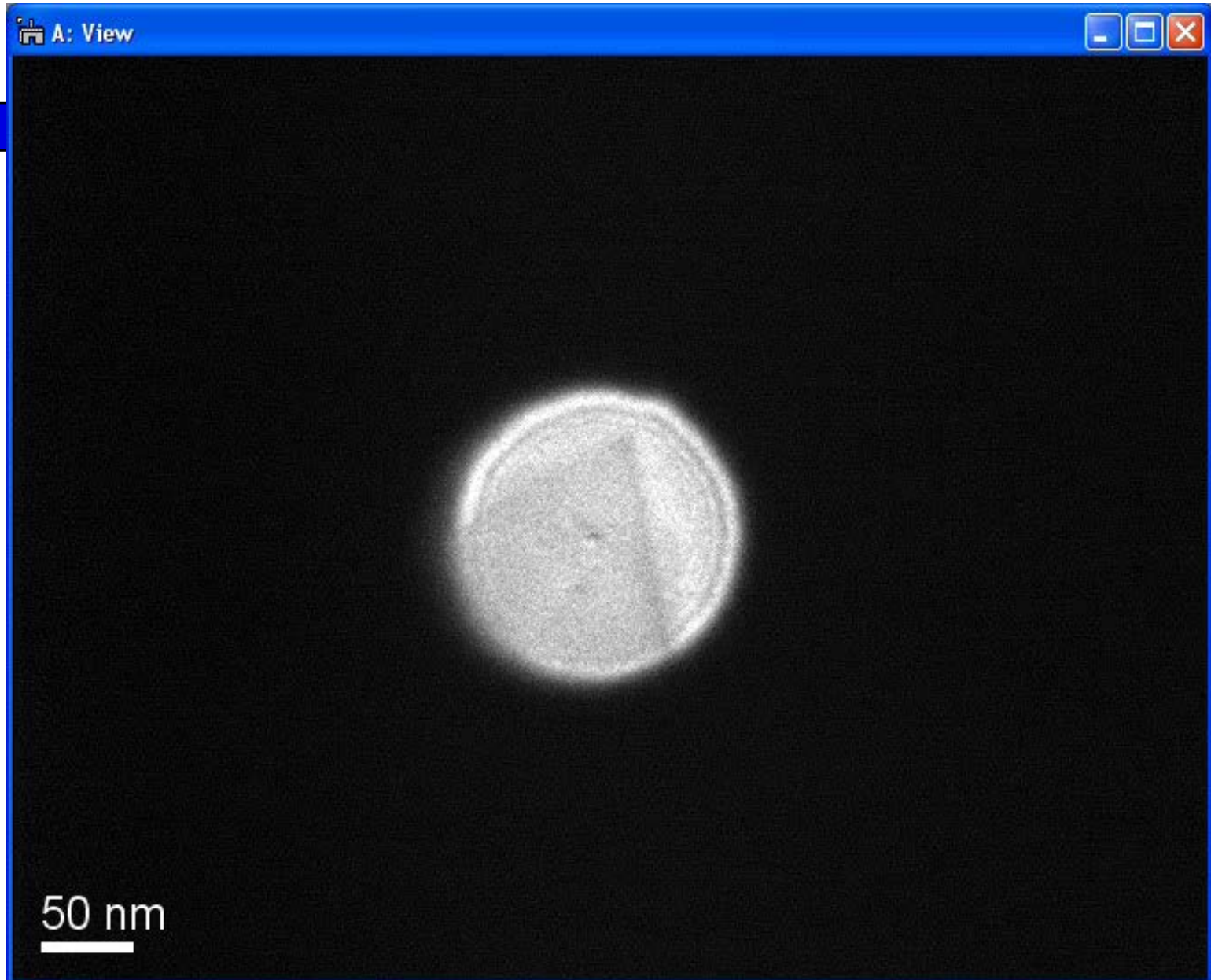
Caveat: this ignores 3-fold astigmatism in pre/post field which is probably not appropriate, and any projector distortions




100 nm




# Alignment can be tricky



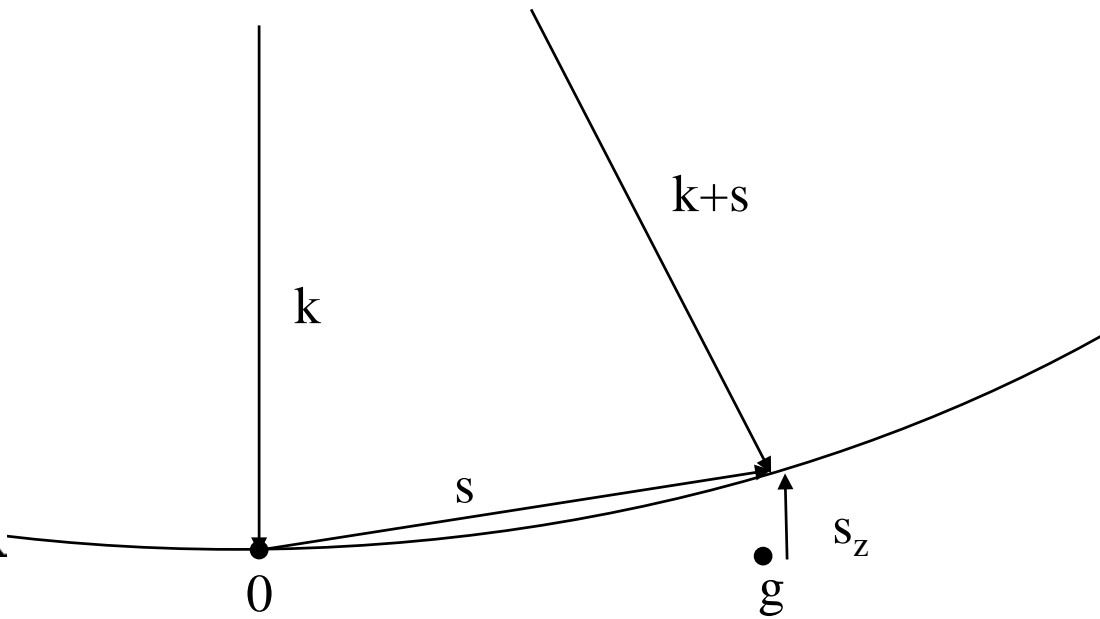
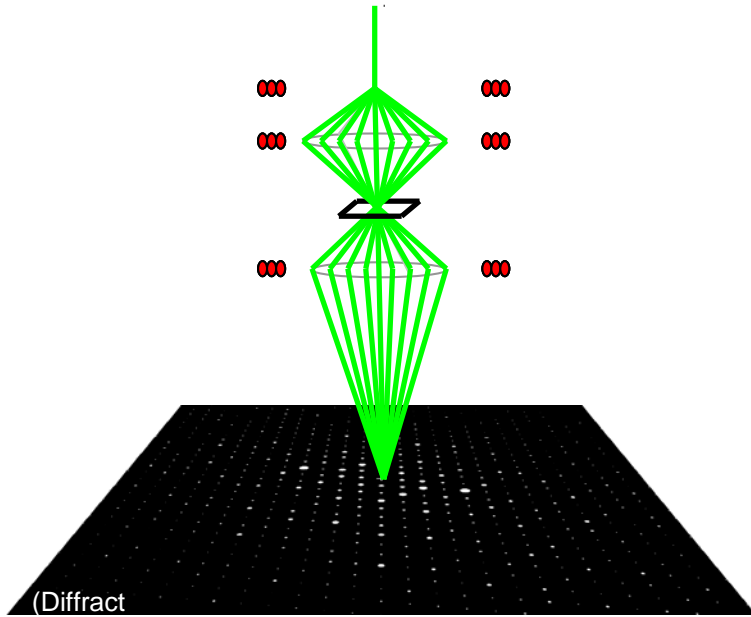
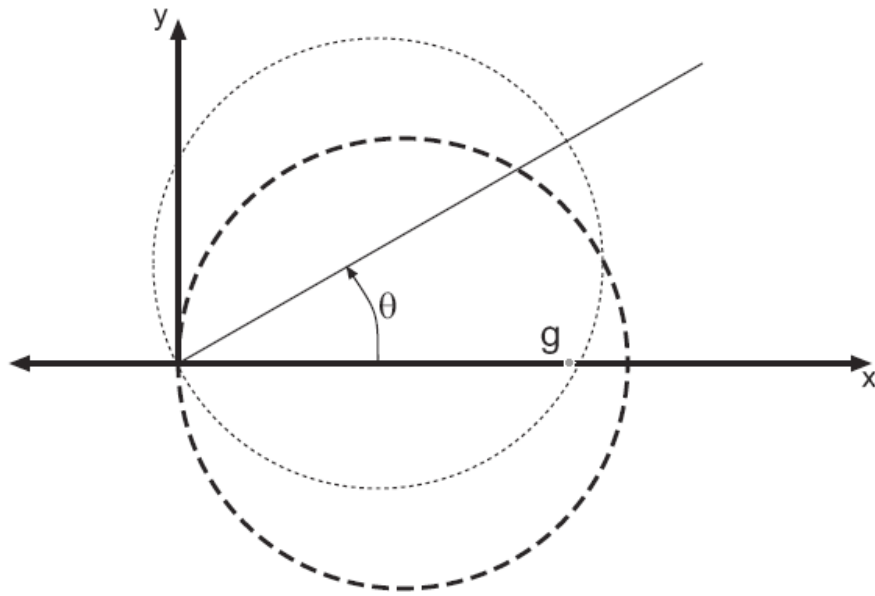
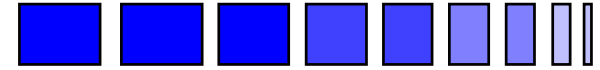
# Why?

- 
- Although PED has been around since 1992, and very actively used for ~10 years (mainly in Europe), there is *no* simple explanation (many have tried and failed)
  - Explanation is a bit rocket science

# Why?


- 
- What, if any generalizations can be made?
  - Role of Precession Angle
    - Systematic Row Limit
  - Importance of integration
    - Phase insensitivity
    - Important for which reflections are used
    - Fast Integration Options

# Ewald Sphere Construction





# Levels of theory

- 
- Precession integrates each beam over  $s_z$
  - Full dynamical theory
    - All reciprocal lattice vectors are coupled and not separable
  - Partial dynamical theory (2-beam)
    - Consider each reciprocal lattice vector dynamically coupled to transmitted beam only
  - Kinematical theory
    - Consider only role of  $s_z$  assuming weak scattering
  - Bragg's Law
    - $I = |F(\mathbf{g})|^2$

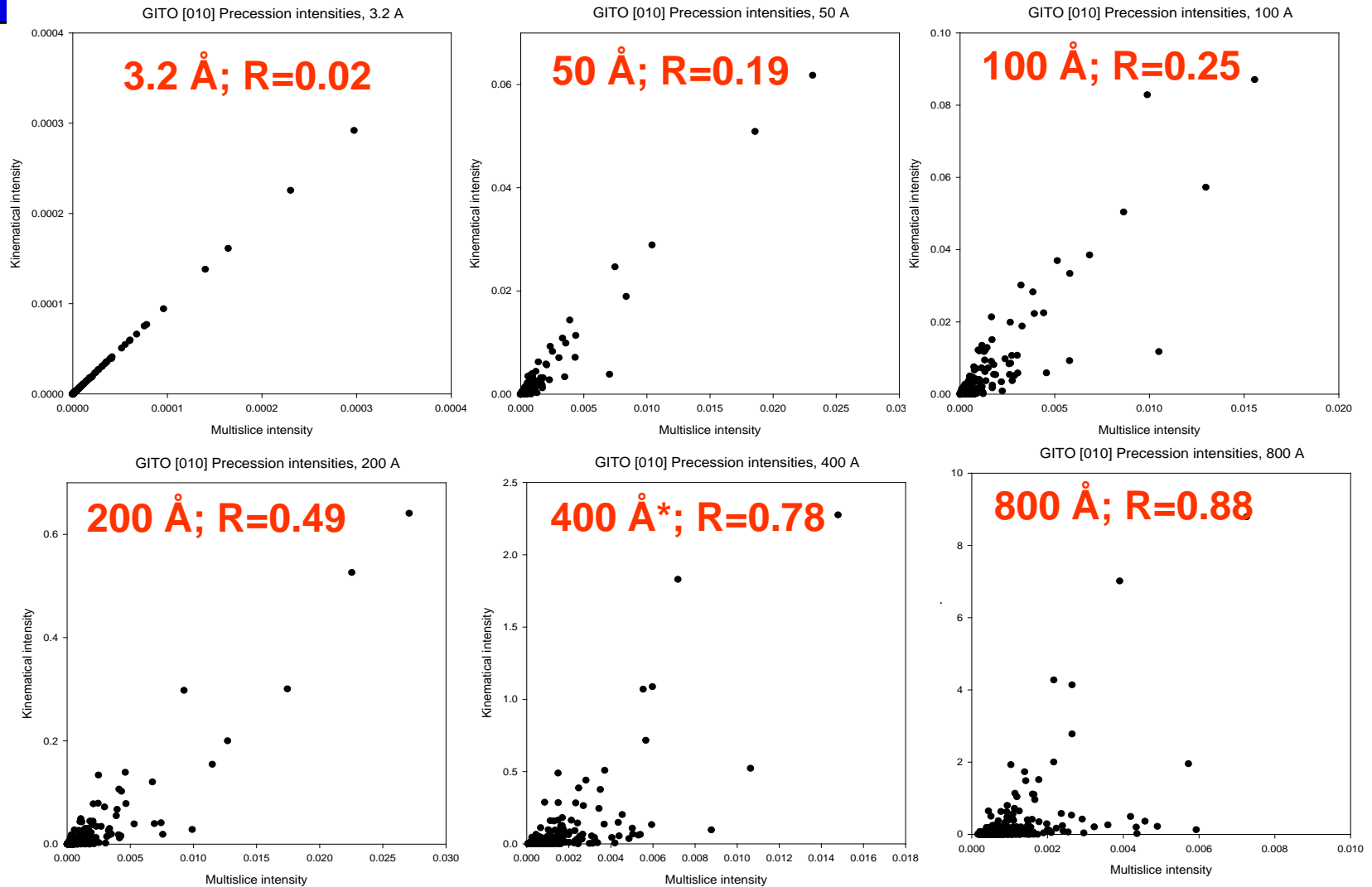
# Early Models

$I_{\text{obs}}$  depends upon  $|F(g)|$ ,  $g$ ,  $\phi$  (precession angle) which we “correct” to the true result

Options:

- 0) No correction at all,  $I=|F(g)|^2$
- 1) Geometry only (Lorentz, by analogy to x-ray diffraction) corresponds to angular integration
- 2) Geometry plus multiplicative term for  $|F(g)|$

# Bragg's Law fails badly $(\text{Ga,In})_2\text{SnO}_5$



# Kinematical Lorentz Correction

$$I(\mathbf{g}) = \int |F(\mathbf{g}) \sin(\pi t s_z) / (\pi s_z)|^2 ds_z$$

$s_z$  taken appropriately over the Precession Circuit

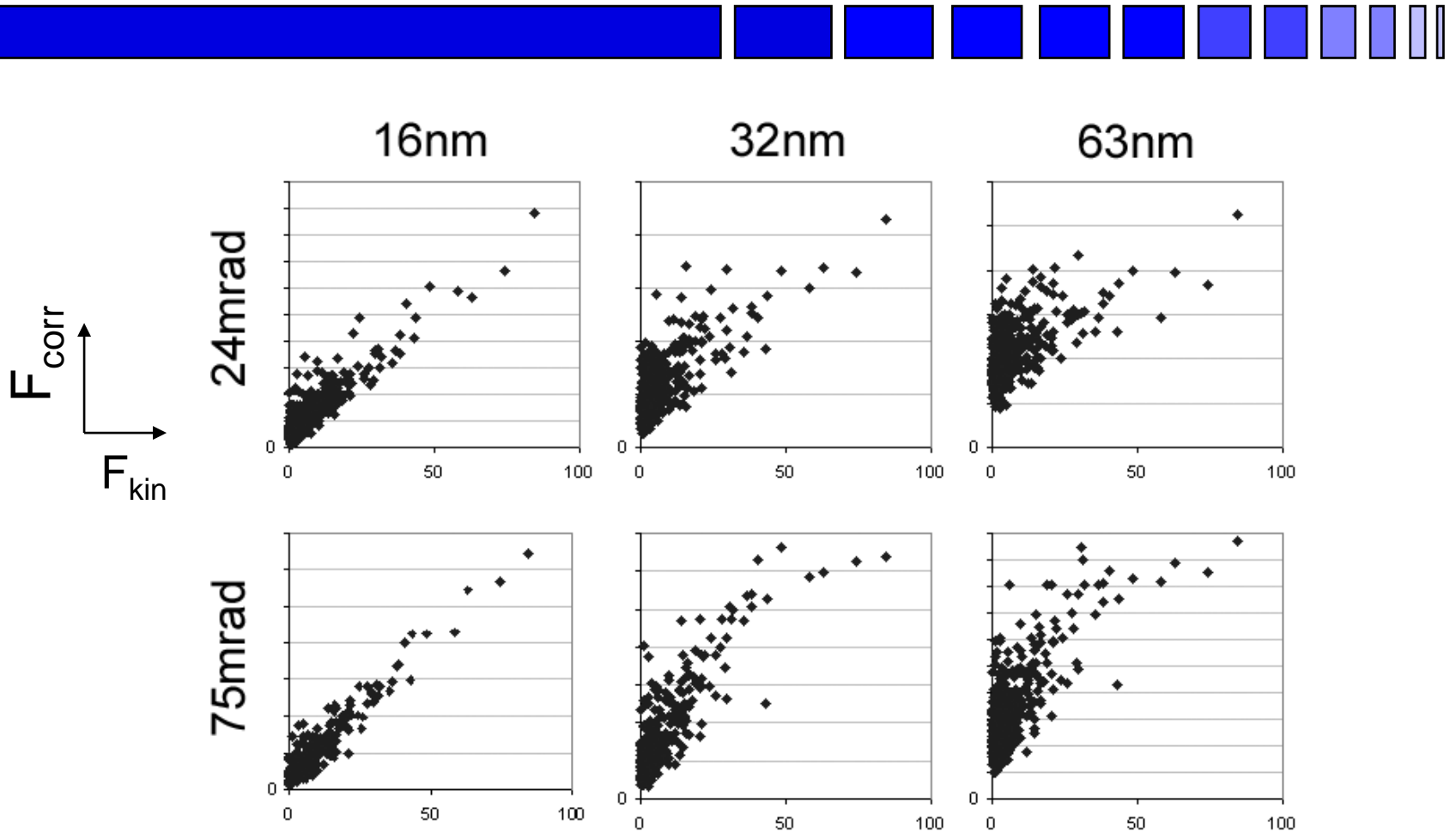
$t$  is crystal thickness (column approximation)

$\phi$  is total precession angle

$$I(\mathbf{g}) = |F(\mathbf{g})|^2 L(\mathbf{g}, t, \phi) \quad L(\mathbf{g}, t, \phi) = g \sqrt{1 - \left(\frac{g}{2R_0}\right)^2}$$

K. Gjønnnes, Ultramicroscopy, 1997.

# Kinematical Lorentz correction: Geometry information is insufficient



Need structure factors to apply the correction!

# 2-Beam (Blackman) form



$$I_{Blackman}(t) = \int_0^{A_g} J_0(2x) dx; \quad A_g(k) \propto tF(k)$$

Limits:

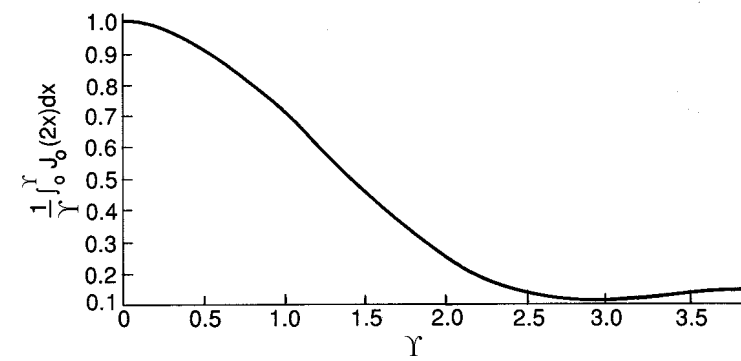
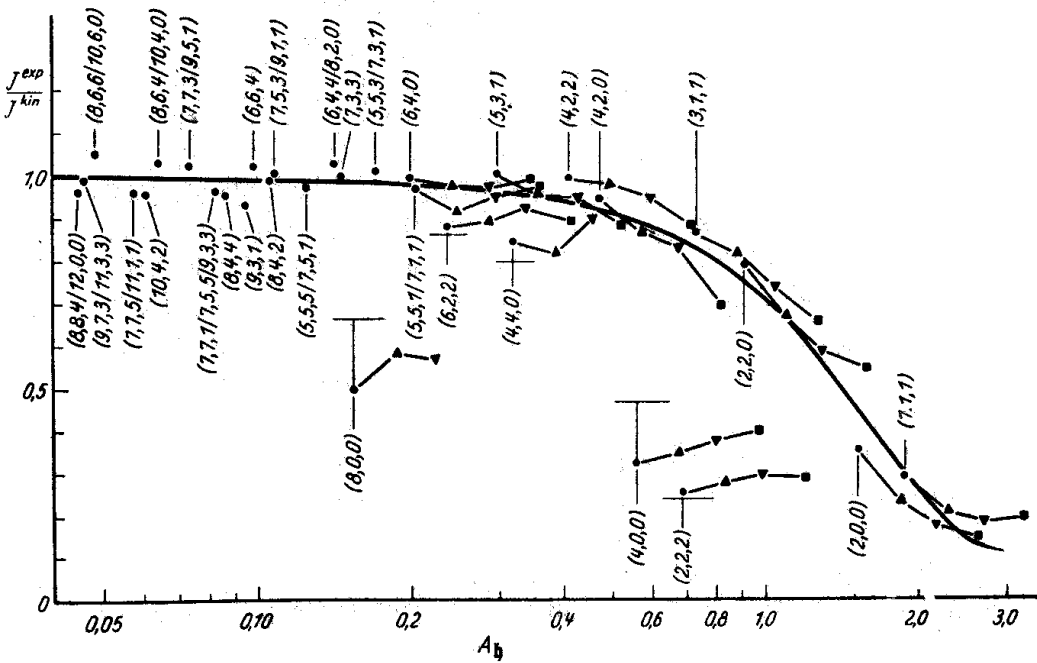
$$A_g \text{ small}; \quad I_{\text{dyn}}(\mathbf{k}) \propto I_{\text{kin}}(\mathbf{k})$$

$$A_g \text{ large}; \quad I_{\text{dyn}}(\mathbf{k}) \propto \sqrt{I_{\text{kin}}(\mathbf{k})} = |F_{\text{kin}}(\mathbf{k})|$$

But...

This assumes integration over all angles,  
which is not correct for precession (correct  
for powder diffraction)

# Blackman Form



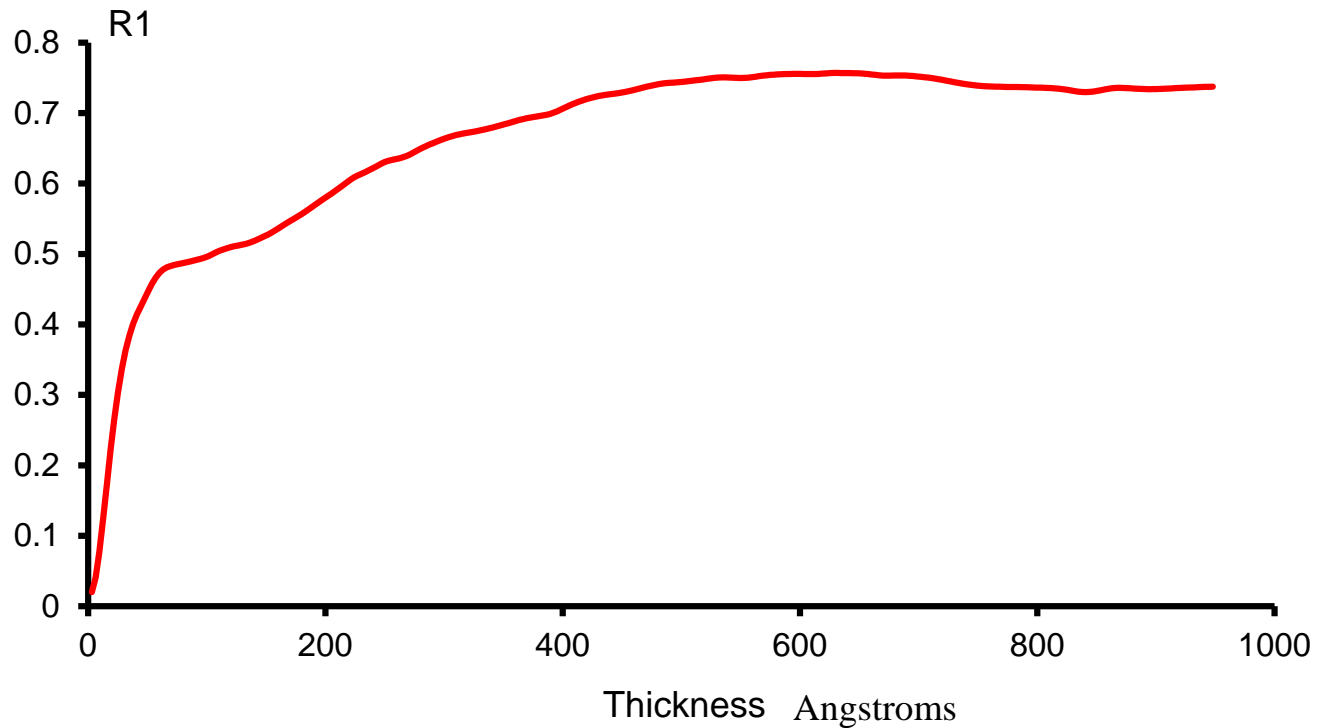
"Blackman curve" used for "two-beams corrections"

The Blackman curve (Blackman [1939]) for the ratio of dynamical to kinematical intensities for a ring pattern as a function of  $A = \sigma H \Phi_h$ . The experimental points are those obtained by Horstmann and Meyer [1965], from measurements on ring patterns from aluminum films at various voltages. The short horizontal lines indicate values calculated using the Bethe potentials, equation (12). (After Horstmann and Meyer [1965].)

# Blackman+Lorentz



Comparison with full calculation, 24 mRad



Alas, little better than kinematical



# Two-Beam Form

$$I(\mathbf{g}) = \int | F(\mathbf{g}) \sin(\pi t s_z^{\text{eff}}) / (\pi s_z^{\text{eff}}) |^2 ds_z$$

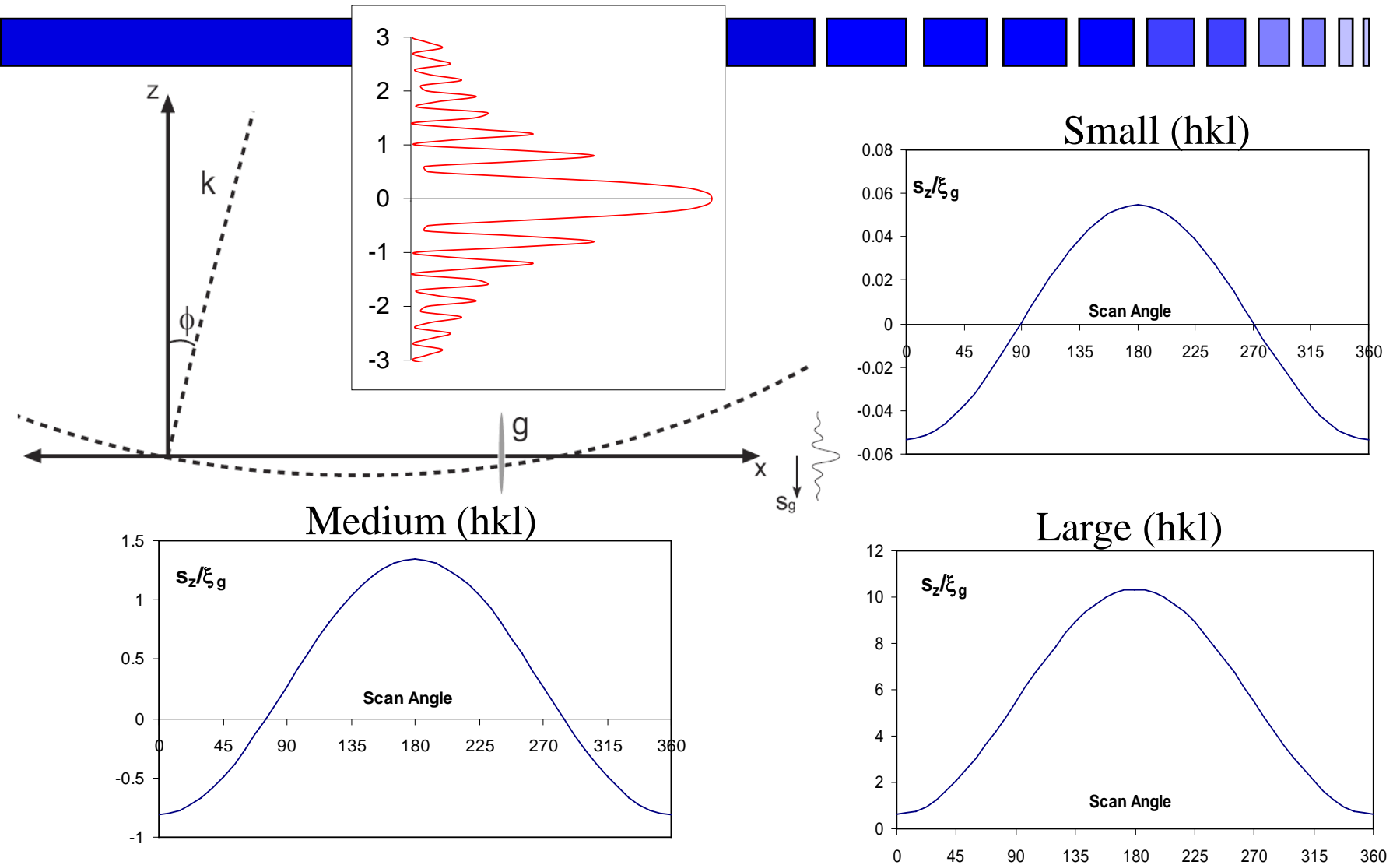
$s_z$  taken appropriately over the Precession Circuit

$$s_z^{\text{eff}} = (s_z^2 + 1/\xi_g^2)^{1/2}$$

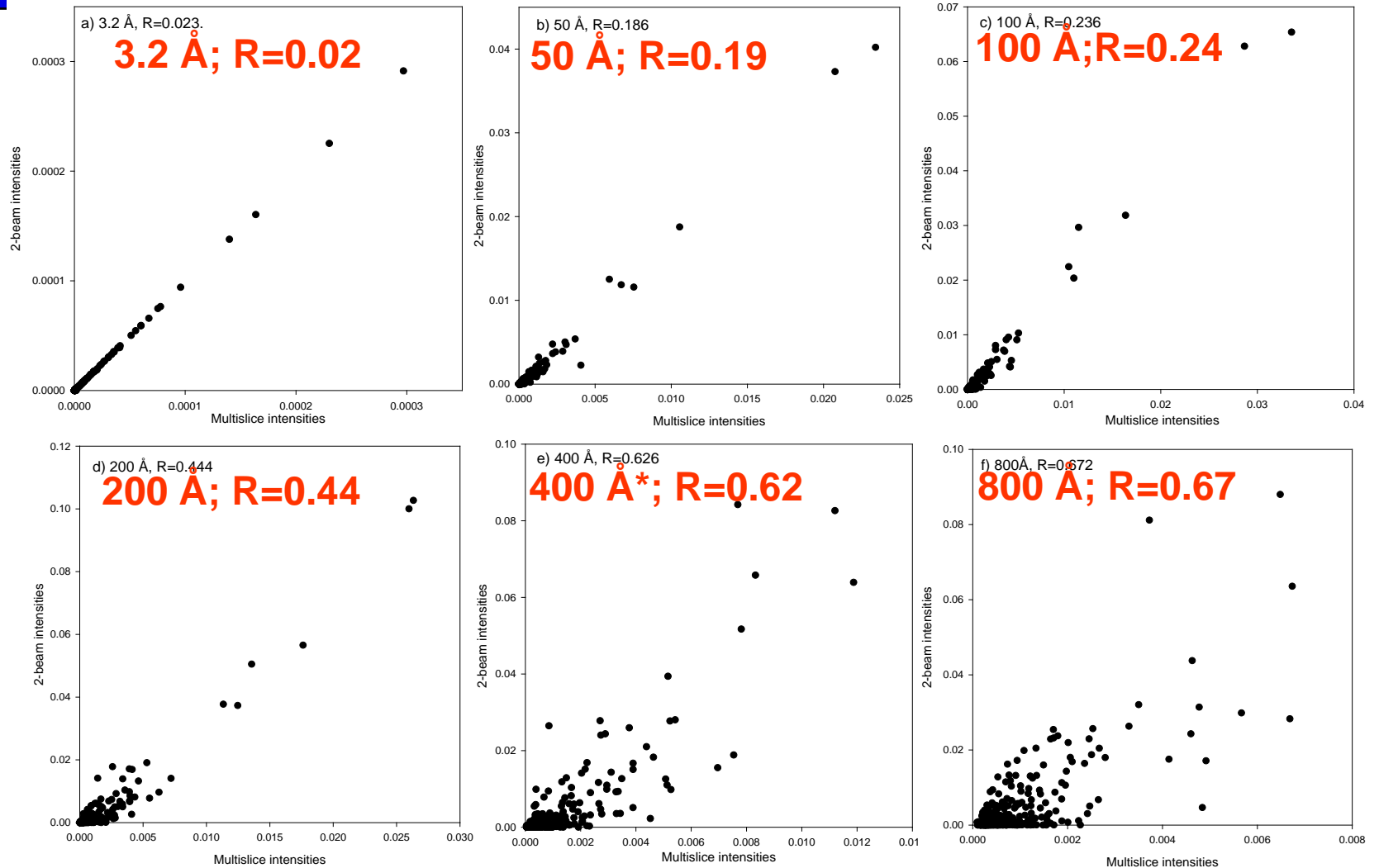
$$\xi_g = \frac{\pi V_c \cos \theta_B}{\lambda F_g}$$

Do the proper integration over  $s_z$

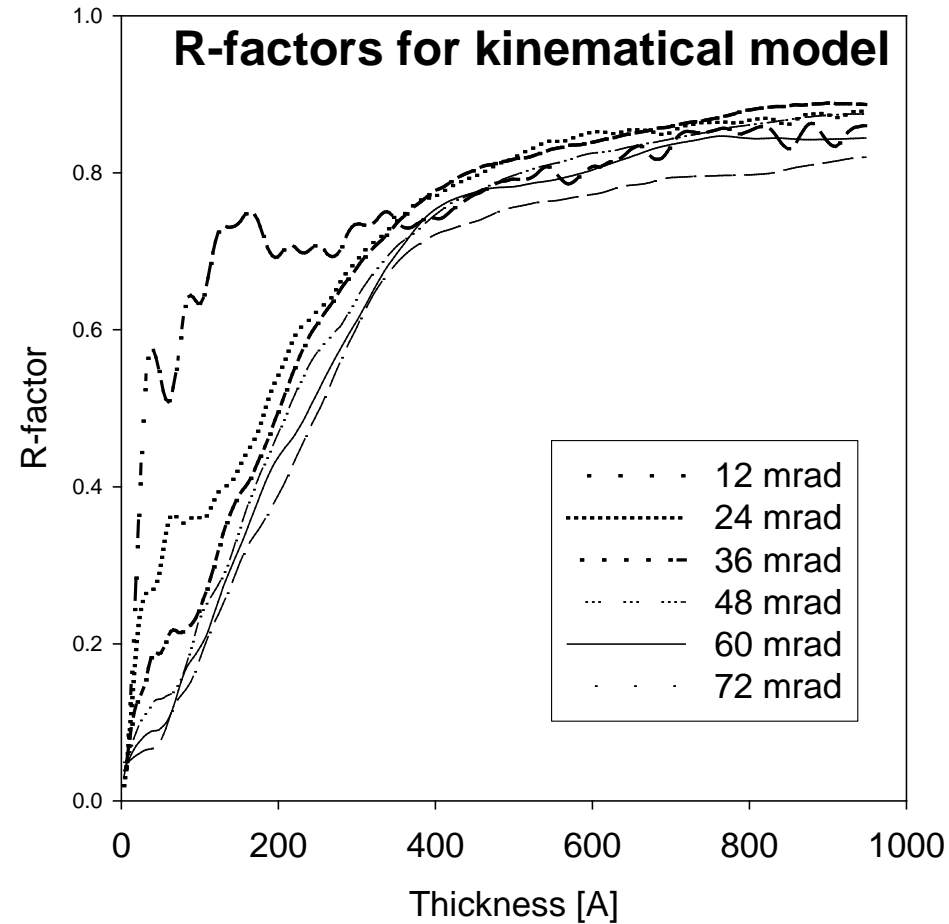
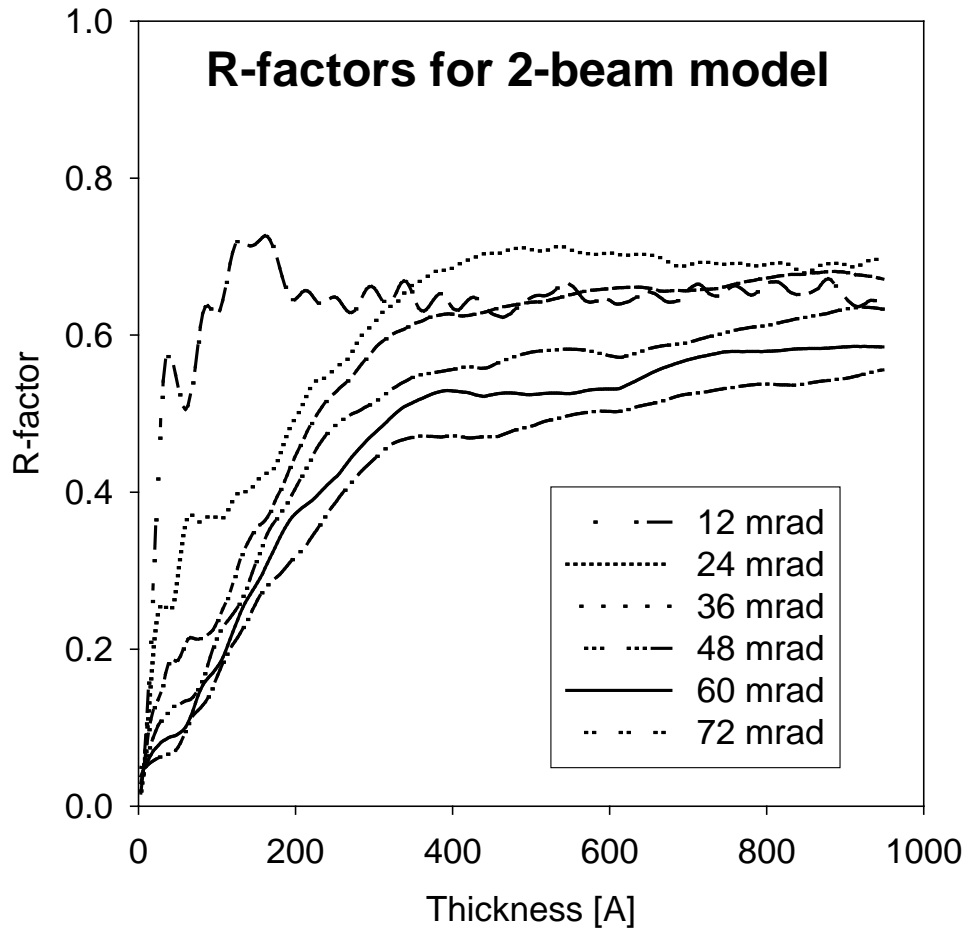
# Two-Beam Integration: Ewald Sphere



# 2-Beam Integration better

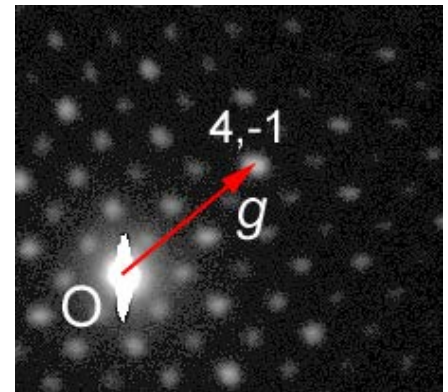
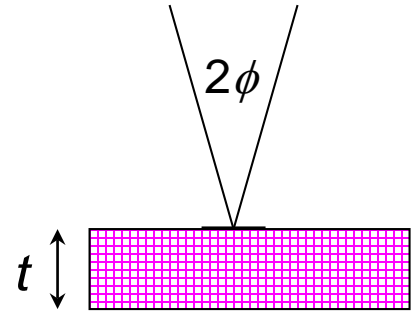


# Some numbers

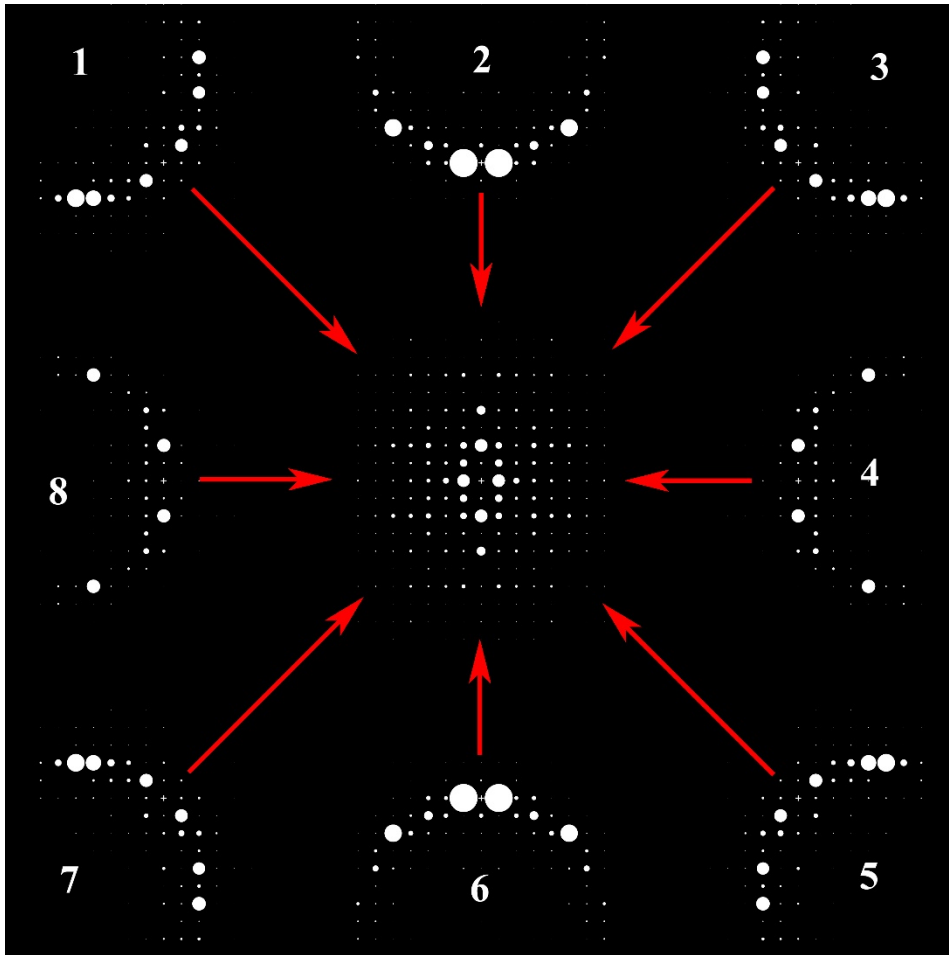


# Fully Dynamical: Multislice

- “Conventional” multislice (NUMIS code, on cvs)
- Integrate over different incident directions  
100-1000 tilts
- $\phi$  = cone semi-angle
  - 0 – 50 mrad typical
- $t$  = thickness
  - ~20 – 50 nm typical
  - Explore: 4 – 150 nm
- $\mathbf{g}$  = reflection vector
  - $|\mathbf{g}| = 0.25 - 1 \text{ \AA}^{-1}$  are structure-defining



# Multislice Simulation

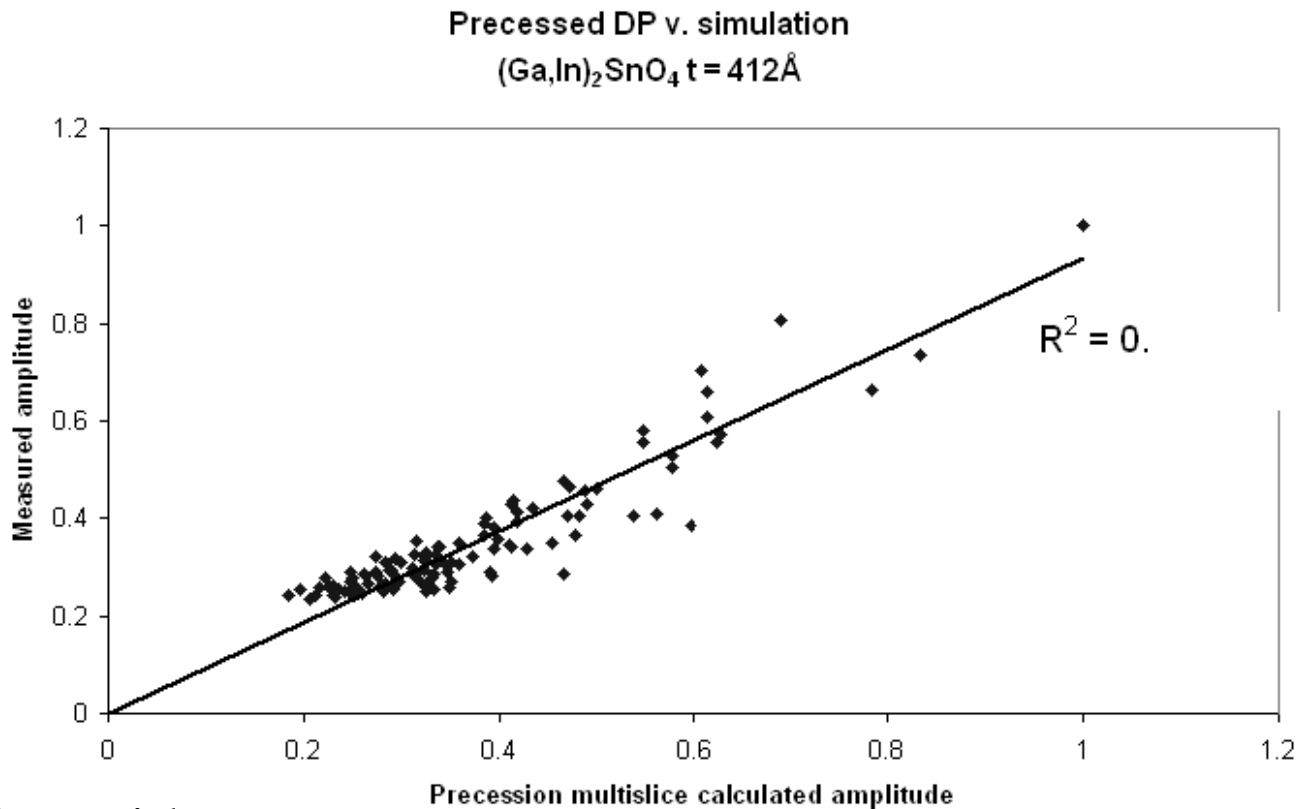


Multislice simulations carried out using 1000 discrete tilts (8 shown) incoherently summed to produce the precession pattern<sup>1</sup>

How to treat scattering?

- 1) Doyle-Turner (atomistic)
- 2) Full charge density string potential -- later

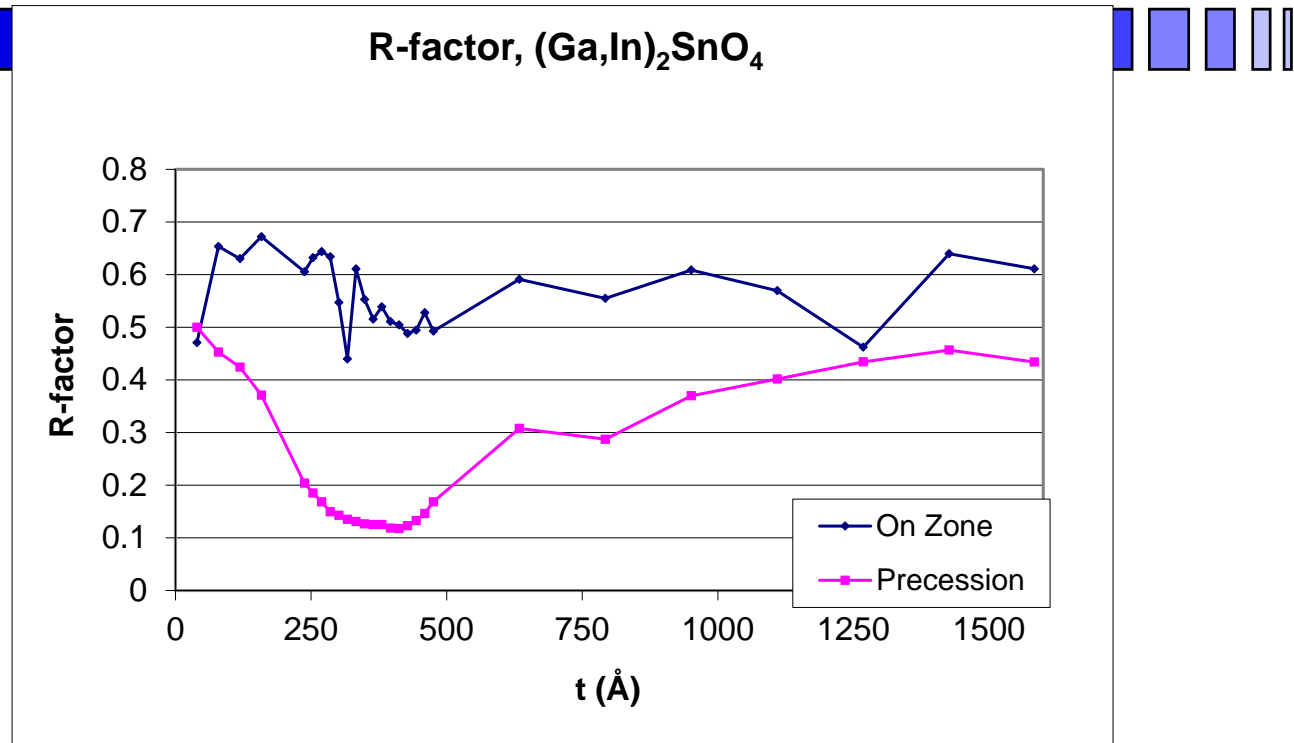
# Multislice Simulation: works (of course)



R1 ~10% without  
refinement of anything

# Global error metric: $R_1$

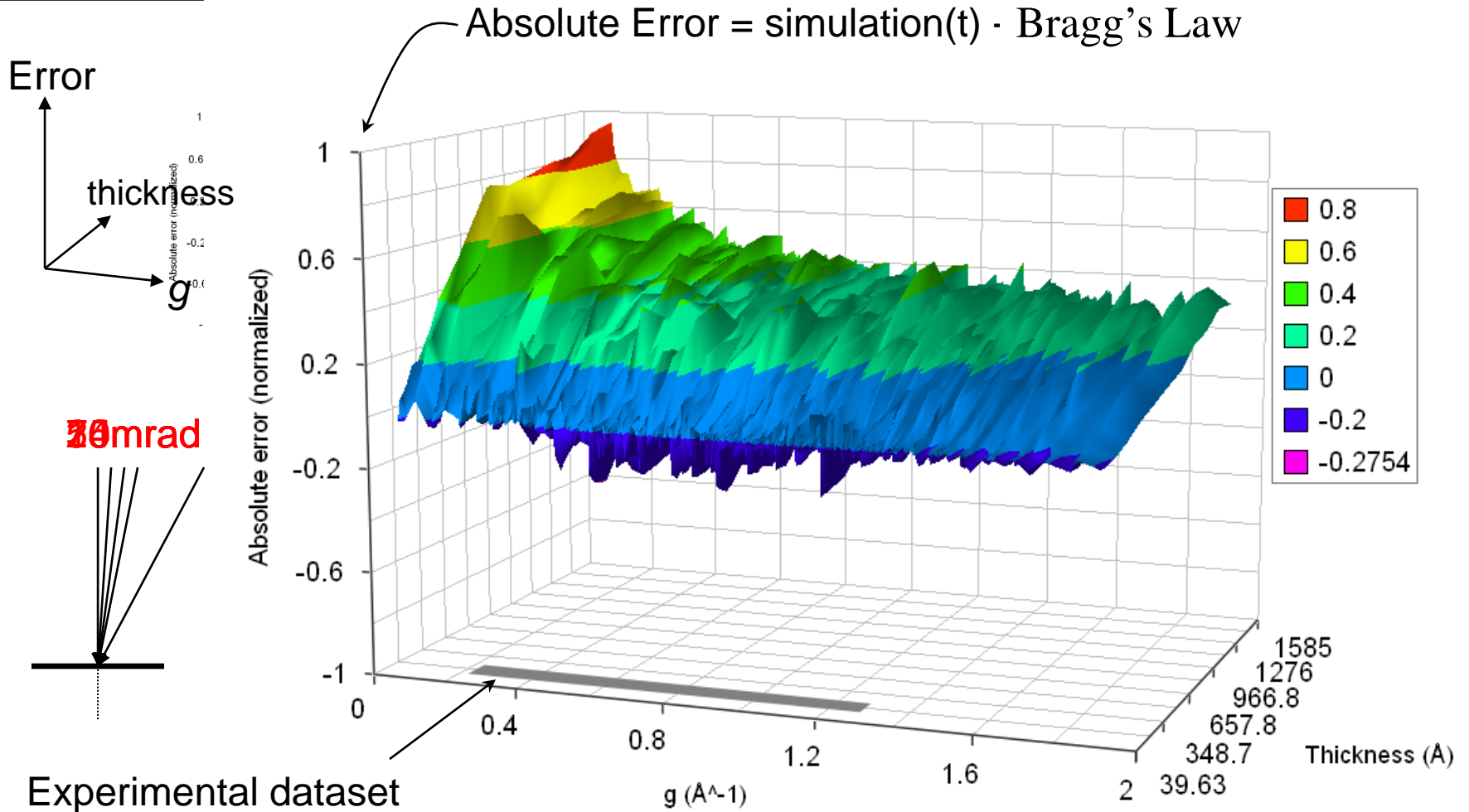
$$R_1 = \frac{\sum |F_{\text{exp}} - F_{\text{sim}}|}{\sum F_{\text{exp}}}$$




- Broad clear global minimum – atom positions fixed
- R-factor = **11.8%** (experiment matches simulated known structure)
  - Compared to >30% from previous precession studies
- Accurate thickness determination:
  - **Average  $t \sim 41\text{nm}$**  (very thick crystal for studying this material)




# Quantitative Benchmark: Multislice Simulation



# Partial Conclusions

- 
- Separable corrections fail; doing nothing is normally better
  - Two-beam correction is not bad (not wonderful)
  - Only correct model is full dynamical one (alas)
  - N.B., Other models, e.g. channelling, so far fail badly – the “right” approximation has not been found

# Overview

- 
- What, if any generalizations can be made?
  - Role of Precession Angle
    - Systematic Row Limit
  - Importance of integration
    - Phase insensitivity
    - Important for which reflections are used

# Role of Angle: Andalusite

## ■ Natural Mineral



□ Orthorhombic (Pnmm)

□  $a=7.7942$

□  $b=7.8985$

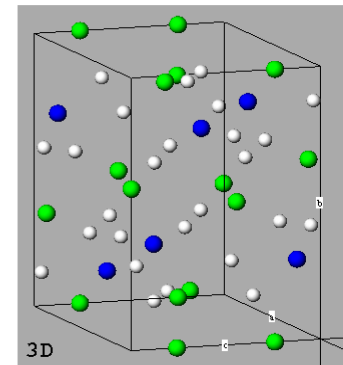
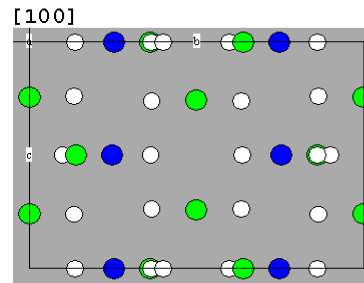
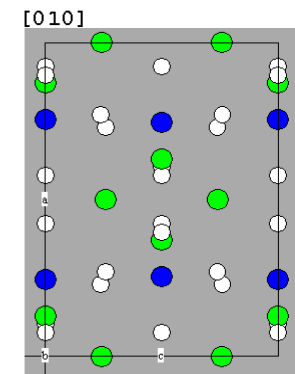
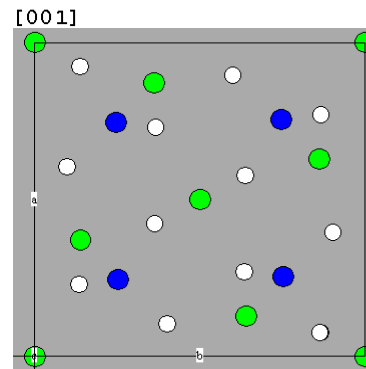
□  $c=5.559$

□ 32 atoms/unit cell

## ■ Sample Prep

□ Crushm Disperse on  
holey carbon film

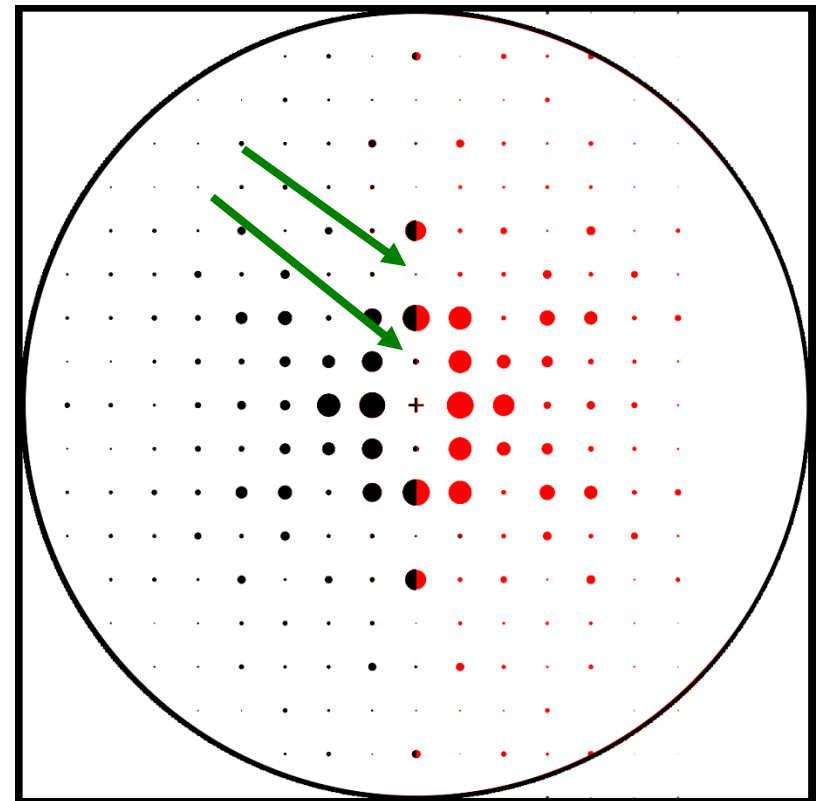
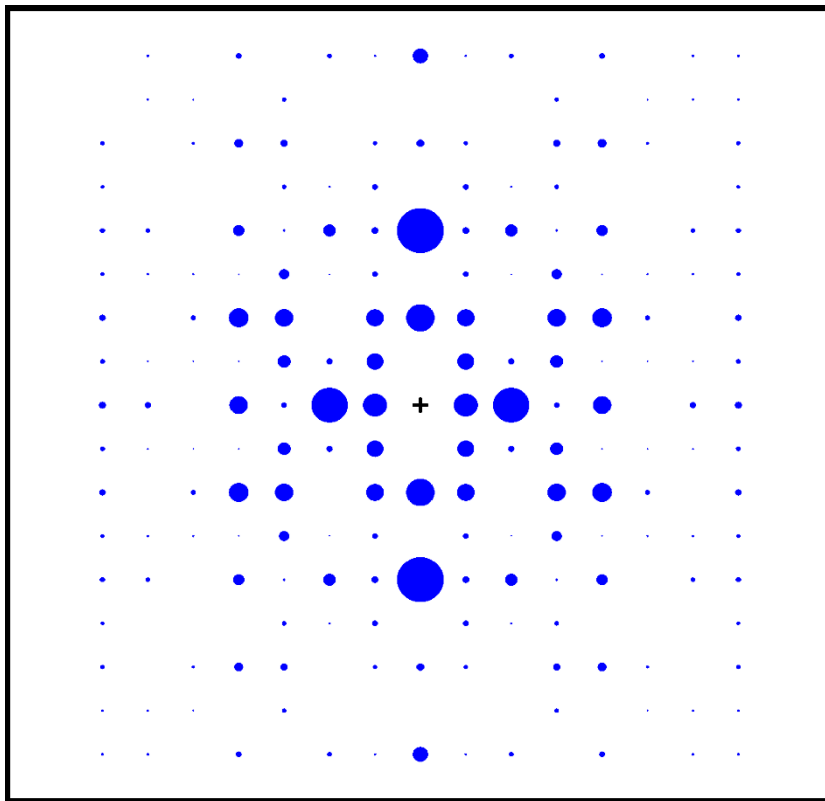
□ Random Orientation



# Measured and Simulated Precession patterns

● Bragg's Law Simulation

● Experimental  
● Multislice



32 mrad

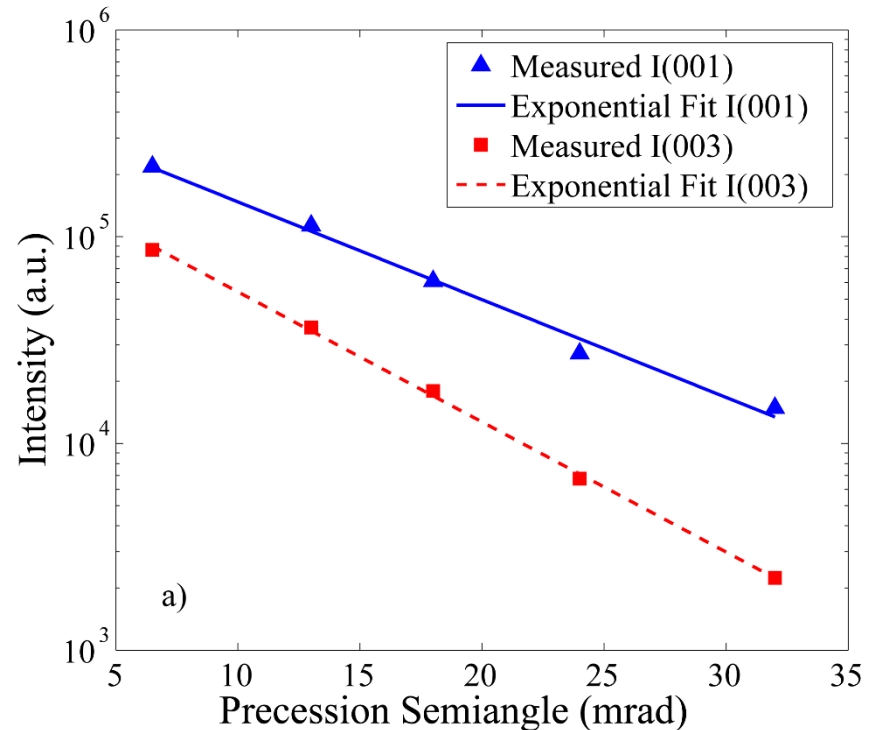
[001]  
[110]

# Decay of Forbidden Reflections

- Decay with increasing precession angle is exponential
  - The non-forbidden (002) reflections decays linearly

$$I = A \exp(-D\phi)$$

	D(001)	D(003)
Experimental	0.109 R <sup>2</sup> =0.991	0.145 R <sup>2</sup> =0.999
Simulated (102nm)	0.112 R <sup>2</sup> =0.986	0.139 R <sup>2</sup> =0.963
Simulated 28-126 nm	0.112±0.012	0.164±0.015



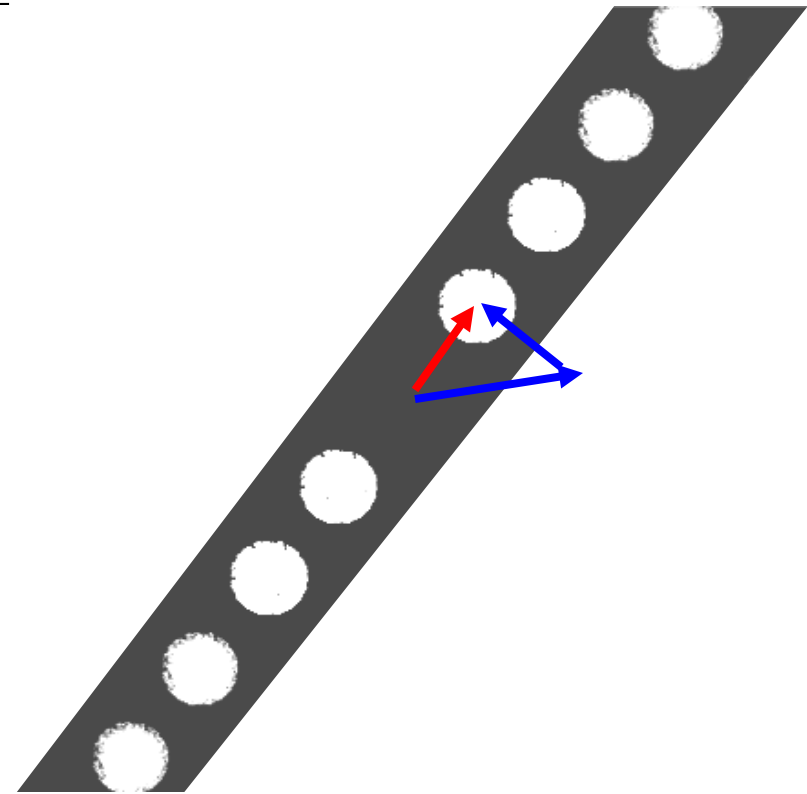
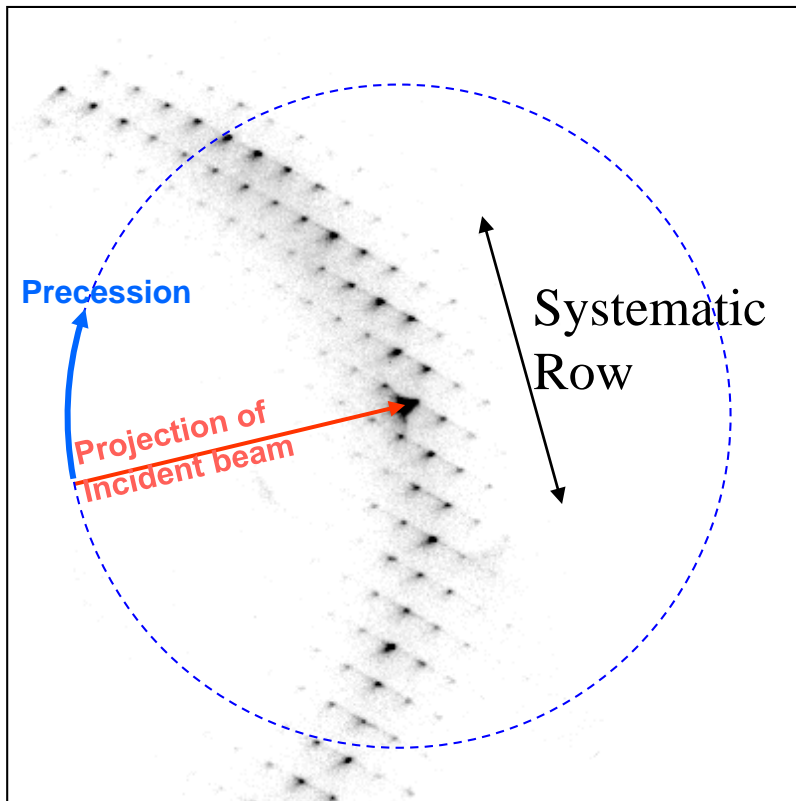
Rate of decay is relatively invariant of the crystal thickness

Slightly different from Jean-Paul's & Paul's – different case

# Quasi-systematic row

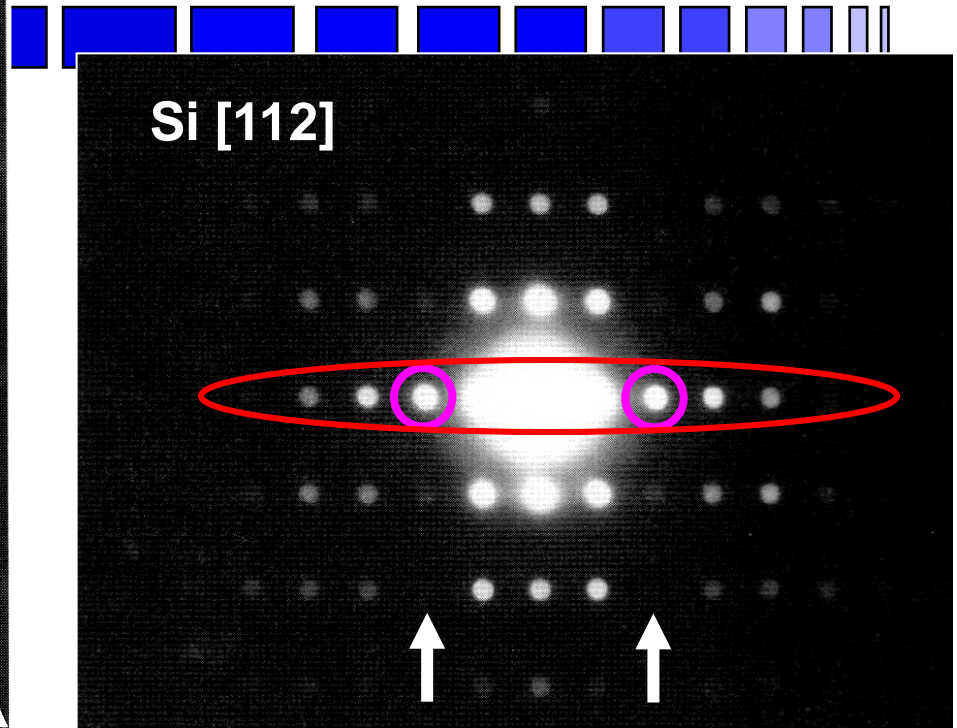
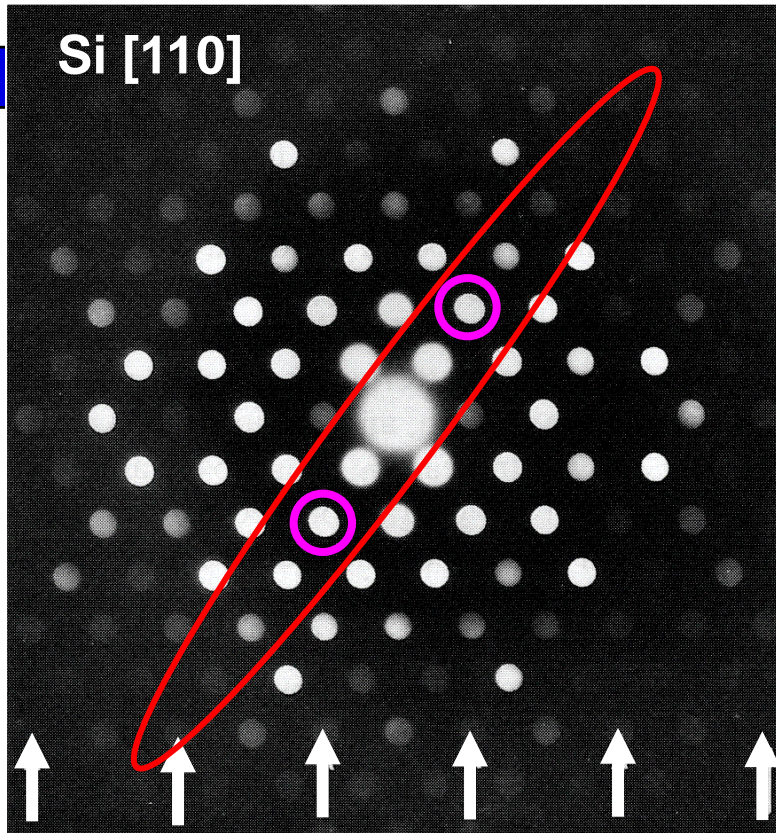


Precessed



**Forbidden, Allowed by Double Diffraction**

# Evidence: 2g allowed



Rows of reflections arrowed should be absent ( $F_{hkl} = 0$ )

Reflections **still present** along strong systematic rows

Known breakdown of Blackman model



# Phase and Averaging

For each of the incident condition generate 50 random phase sets  $\{\varphi\}$  and calculate patterns:

Single settings at 0, 12, 24 and 36 mrad  
(along arbitrary tilt direction).

At fixed semi-angle (36 mrad) average over range of  $\alpha$  as follows:

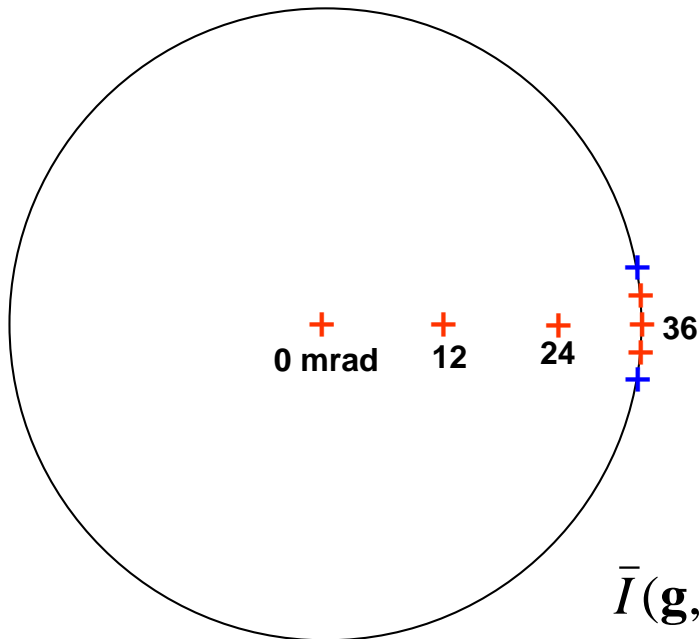
3 settings at intervals of  $0.35^\circ$

5 settings at intervals of  $0.35^\circ$

21 settings at intervals of  $1^\circ$

For each  $\mathbf{g}$  and thickness compute avg., stdev:

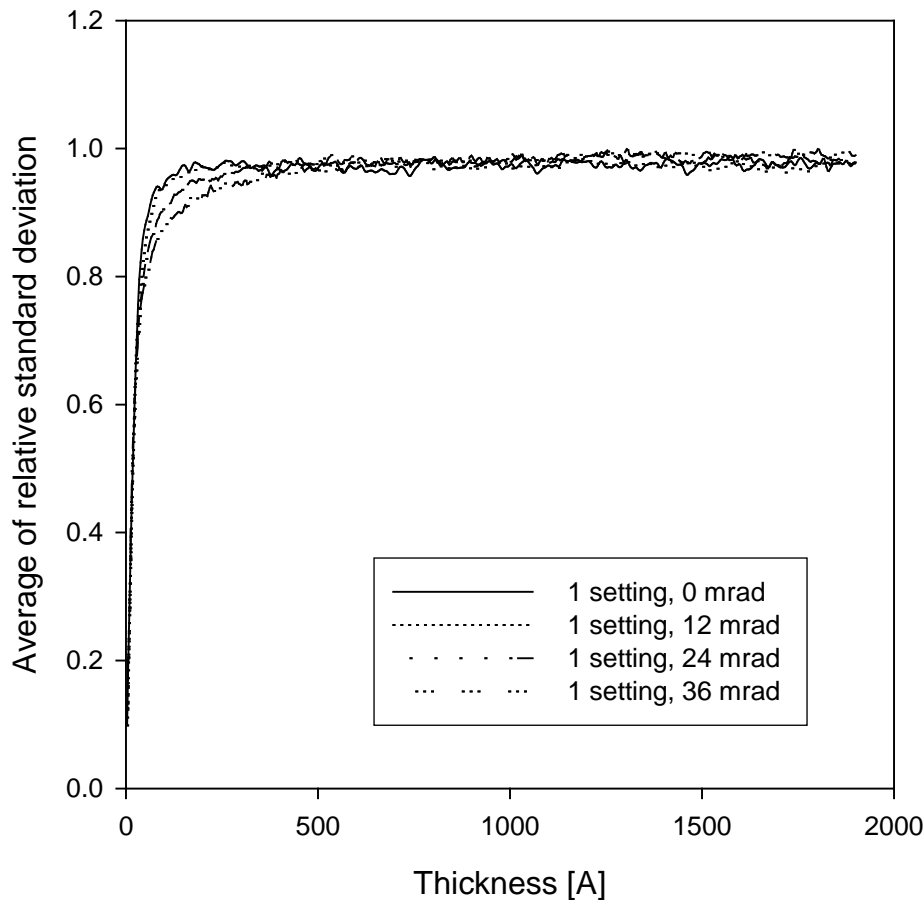
$$\bar{I}(\mathbf{g}, t) = \frac{1}{50} \sum_{\{\varphi\}} I(\mathbf{g}, t) \quad \sigma(\mathbf{g}, t) = \left[ \frac{1}{50} \sum_{\{\varphi\}} I(\mathbf{g}, t)^2 - \bar{I}(\mathbf{g}, t)^2 \right]^{1/2}$$



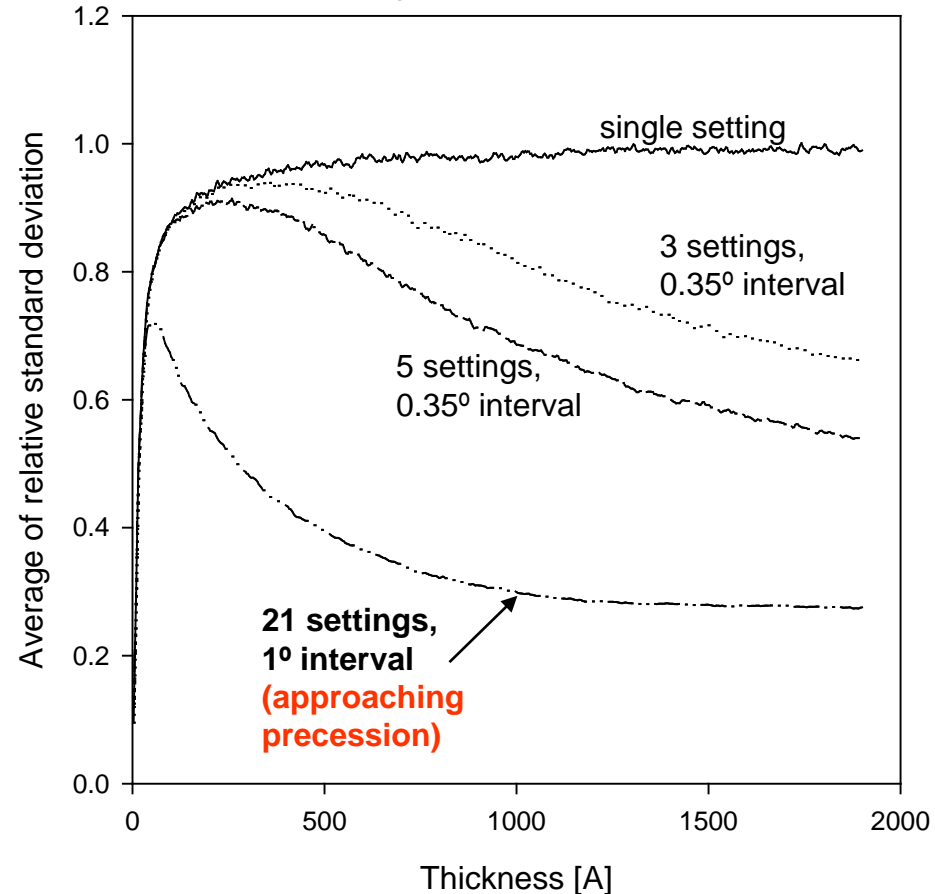
# Phase independence when averaged




Single setting calculations



Multiple settings calculations at  $\phi=36$  mrad

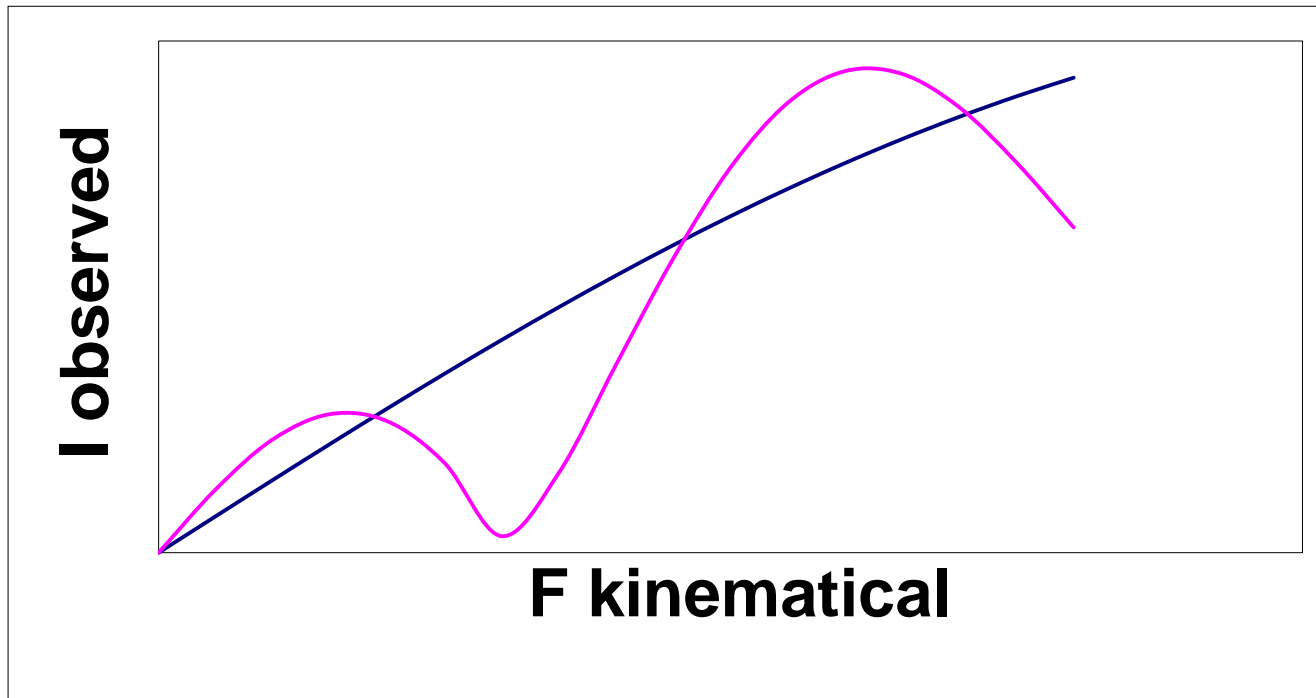


# Why does it work?

- 
- If the experimental Patterson Map is similar to the Bragg's Law Patterson Map, the structure is solvable – Doug Dorset
  - If the deviations from Bragg's Law are statistical and “small enough” the structure is solvable – LDM

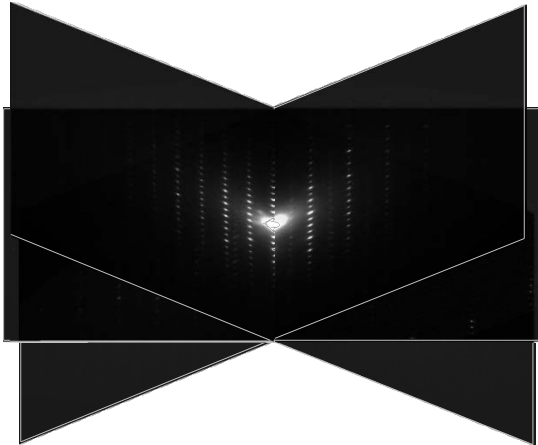
# Why it works - Intensity mapping

- Iff  $I_{\text{obs}}(\mathbf{k}_1) > I_{\text{obs}}(\mathbf{k}_2)$  when  $F_{\text{kin}}(\mathbf{k}_1) > F_{\text{kin}}(\mathbf{k}_2)$ 
  - Structure should be invertible (symbolic logic, triplets, flipping...)

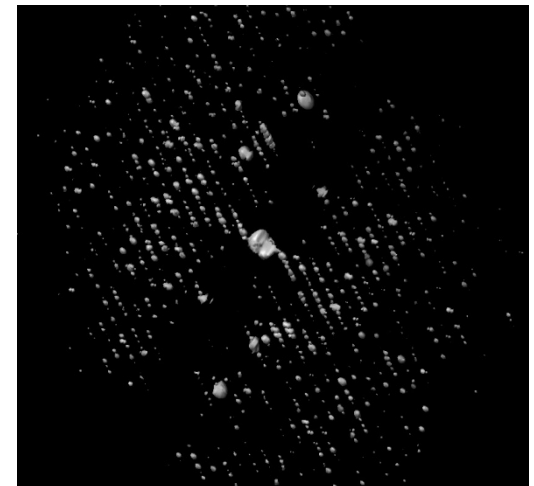
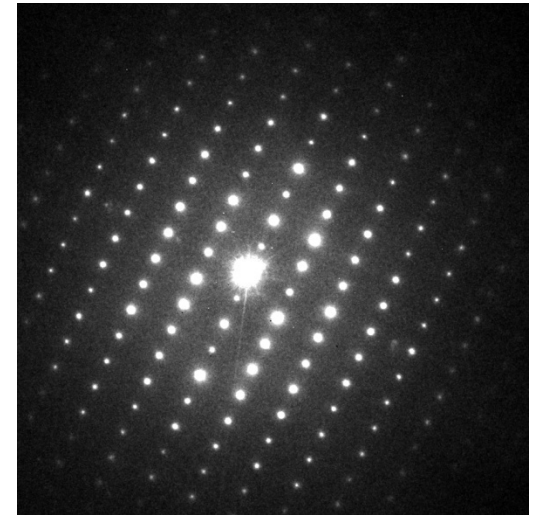


# Automated Diffraction Tomography (ADT)

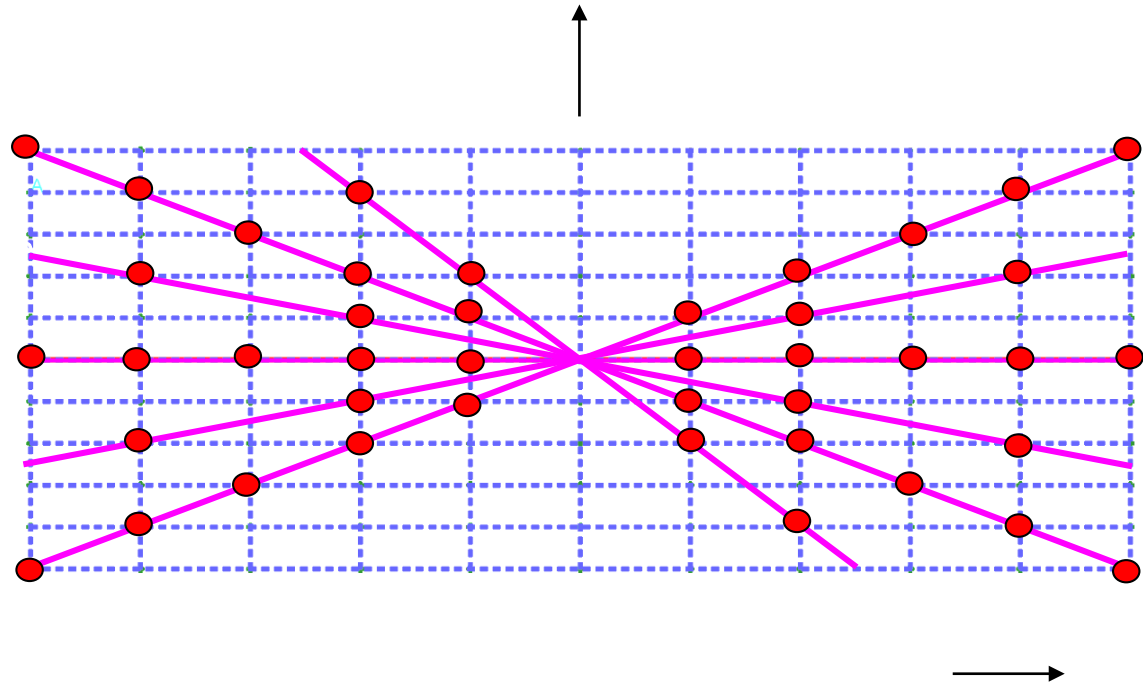
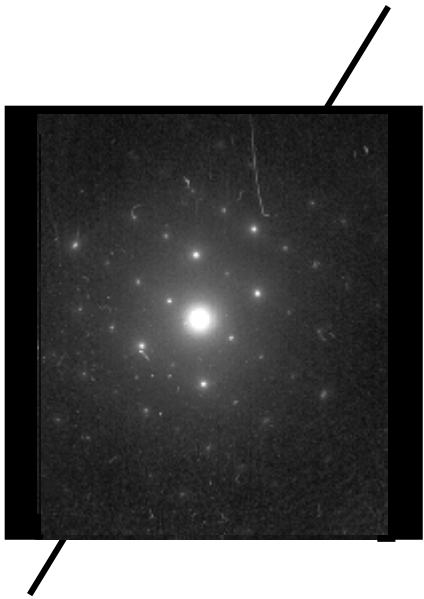
Most electron diffraction use TEM on-zone (oriented) patterns



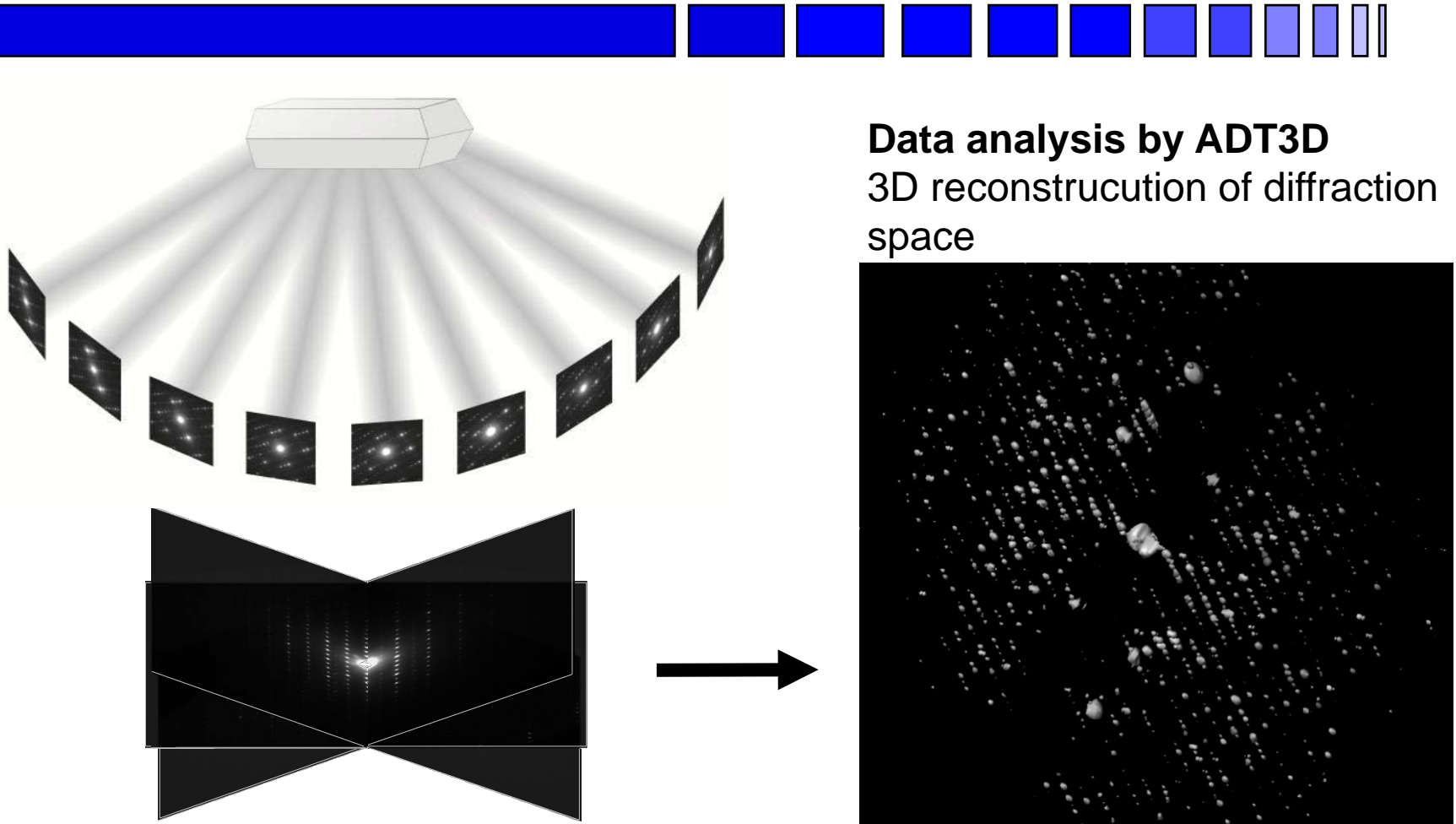
**ADT approach:** collection of full 3D reciprocal space starting from not oriented patterns



# Zone Axis Data



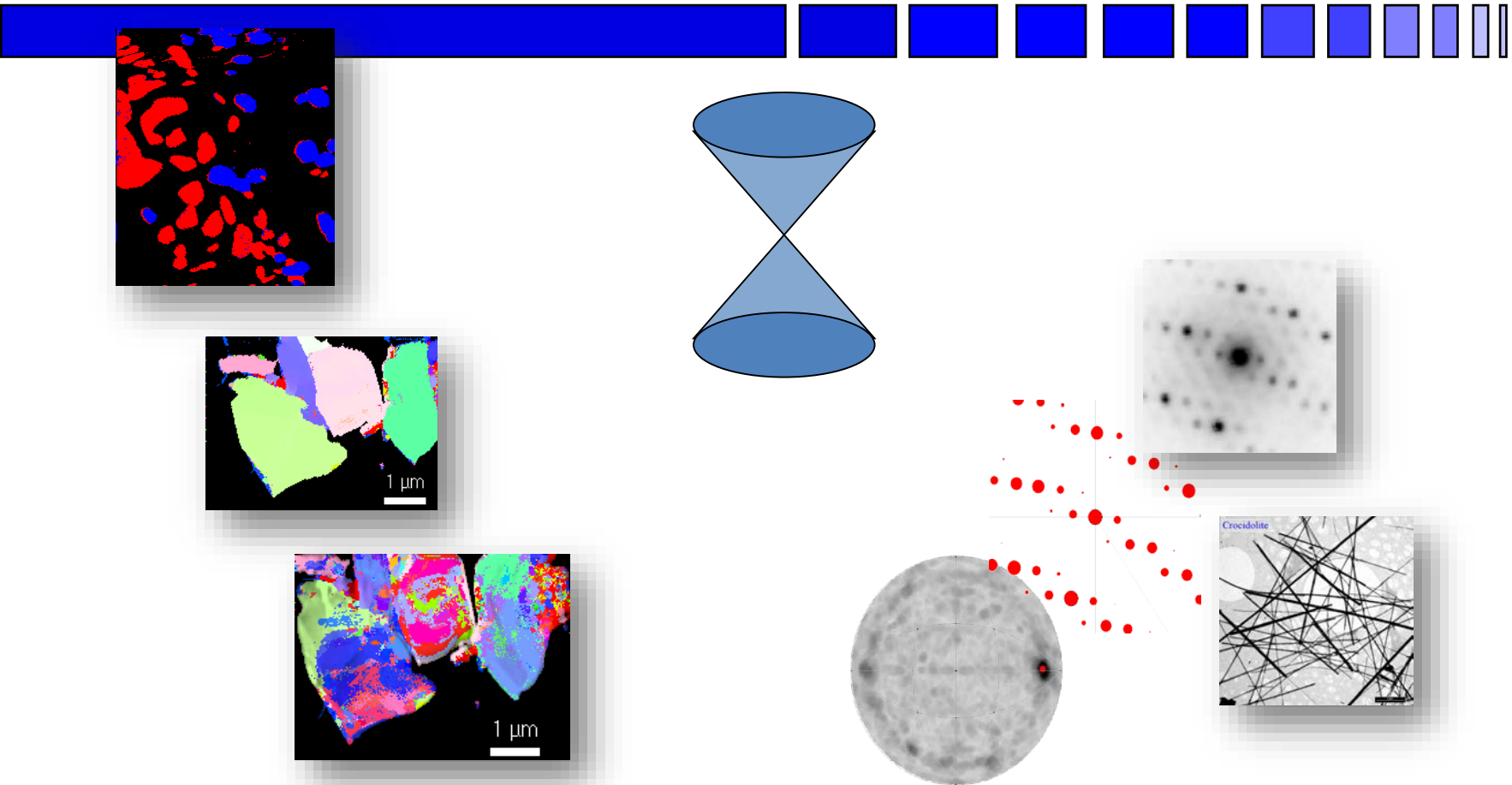
# ADT concept



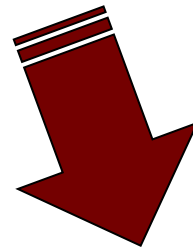
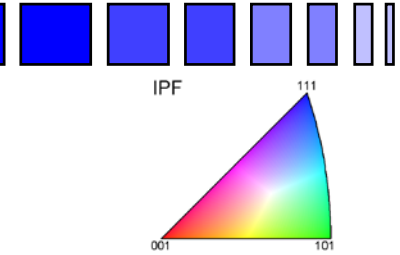
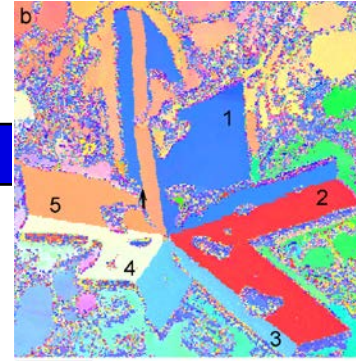
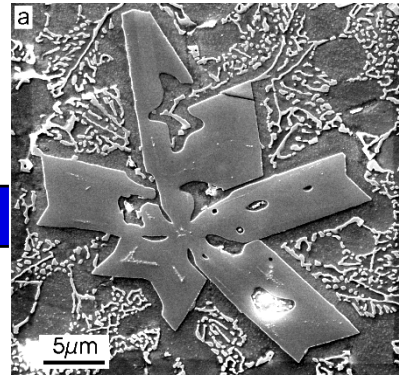
**Data analysis by ADT3D**  
3D reconstruction of diffraction space

# ASTAR (EBSD-TEM like )

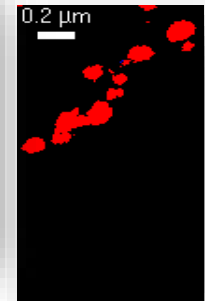
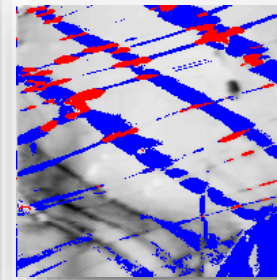
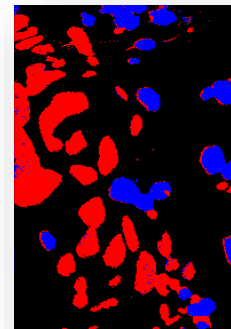
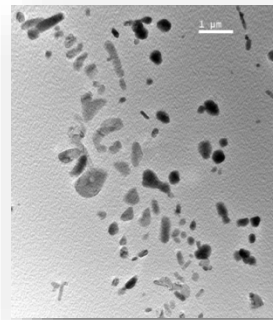
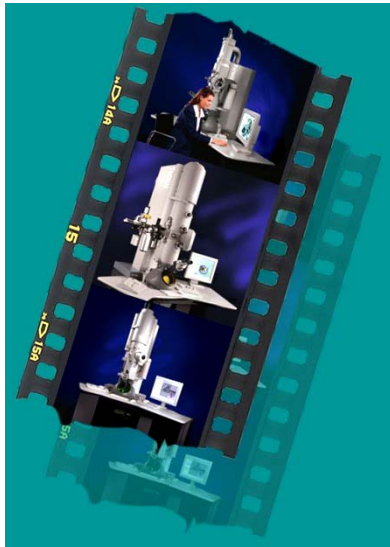
Automatic Crystal Orientation/Phase mapping for TEM







# NEW precession application “EBSD” – TEM

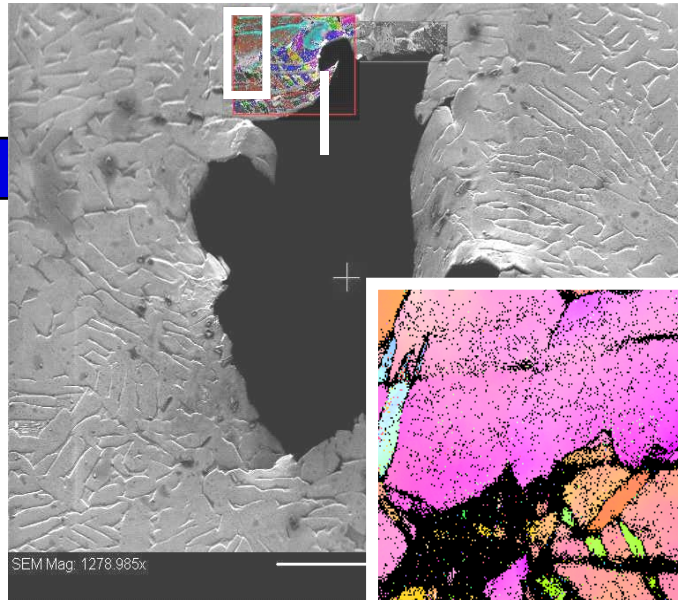


**EBS-TEM : beam is scanned over the sample ( eg.  $10\ \mu \times 10\ \mu$  )**

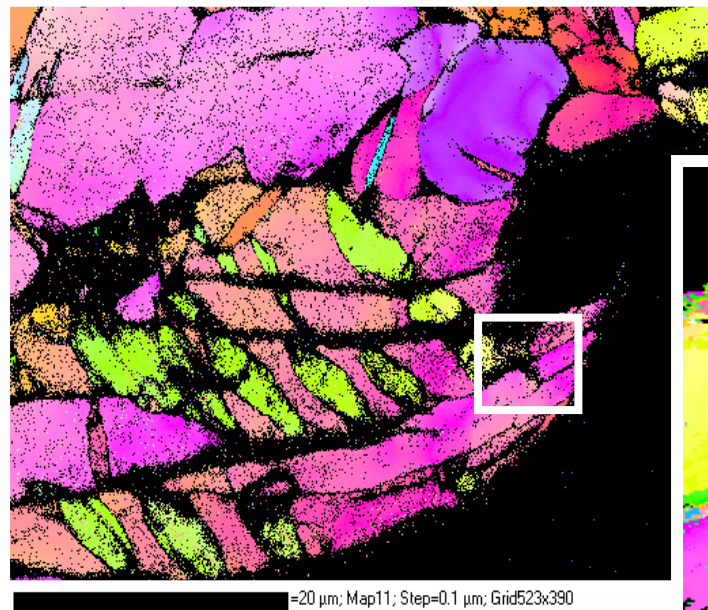
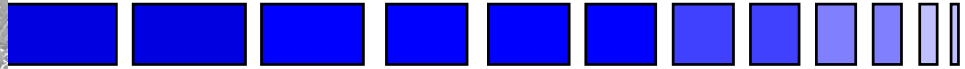


- ➔ **spot electron diffraction patterns are collected ( NOT sensitive to stress/strain or surface sample preparation like in EBSD-SEM )**
- ➔ **Beam scanning performed by “spinning star” unit / no STEM need**
- ➔ **Thousands of experimental spot ED patterns are acquired by a very fast optical CCD camera attached to TEM screen ( 180 patterns/sec )**
- ➔ **Slow scan CCD can also be used ( but slow : 20-30 patterns/sec )**
- ➔ **Thousands of theoretical ED patterns are generated ( templates ) from .cif files or commercial databases for all known phases in a sample**
- ➔ **Template matching is used ( by cross-correlation of all experimental ED patterns with all templates ) to generate most probable orientation of every scanned position in the sample.**

# Comparison SEM-(EBSD) vs TEM spatial resolution

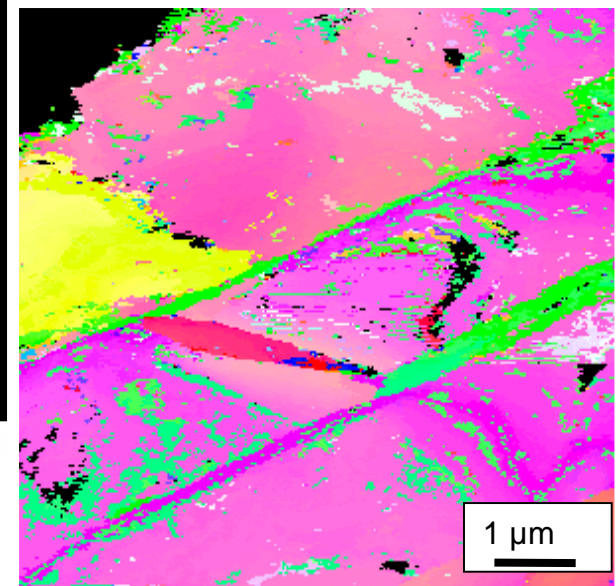


SEM orientation map  
deformed Ta<sub>6</sub>V alloy



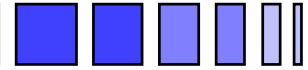
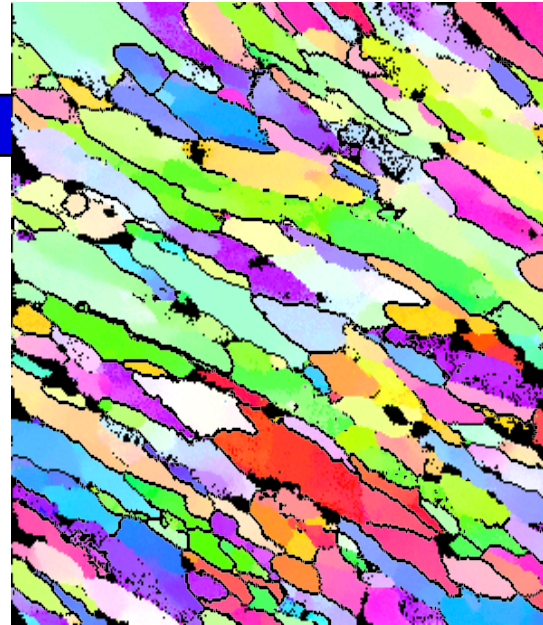
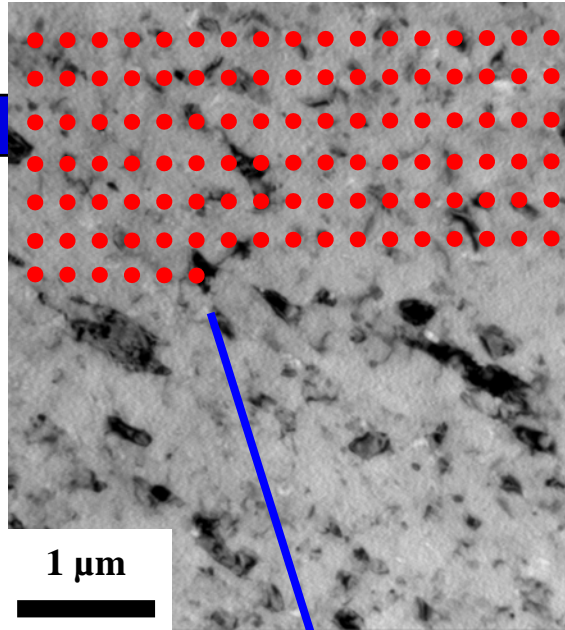
EBSD map (100 nm stepsize)

TEM orientation map  
(25 nm stepsize)

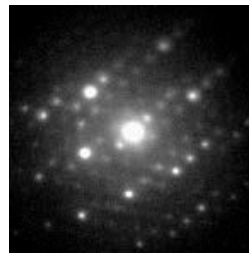




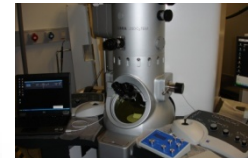
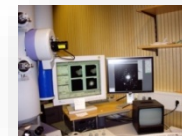
# ASTAR : diffraction pattern adquisition



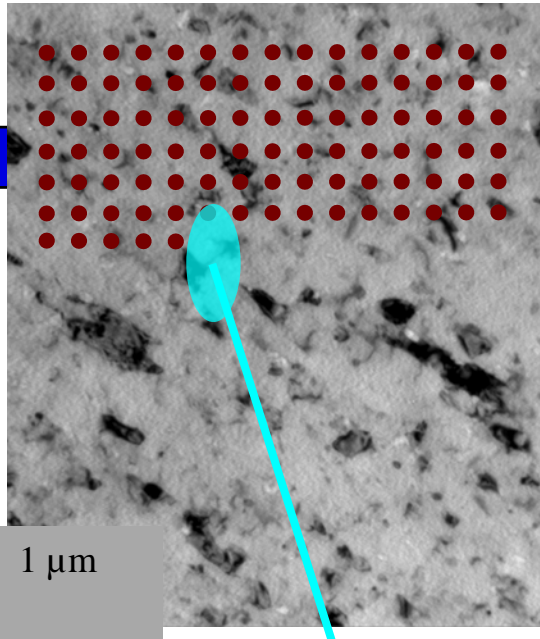
Example :Severely  
deformed  
7075 Aluminium Alloy



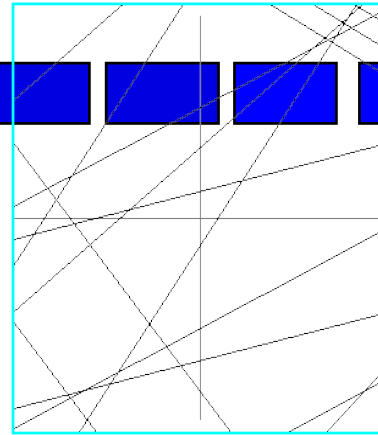
Any TEM -FEG/LaB6  
may work with ASTAR



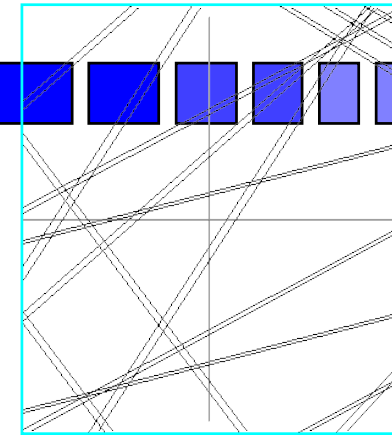
# EBSD-TEM : Automated Crystal Orientation Mapping



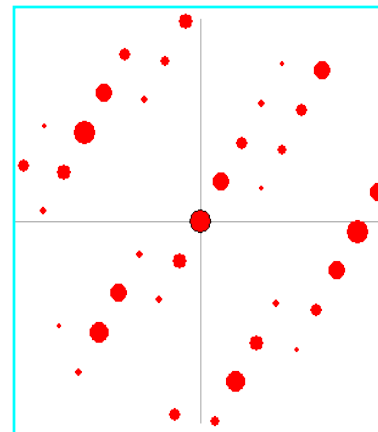
Kikuchi pattern



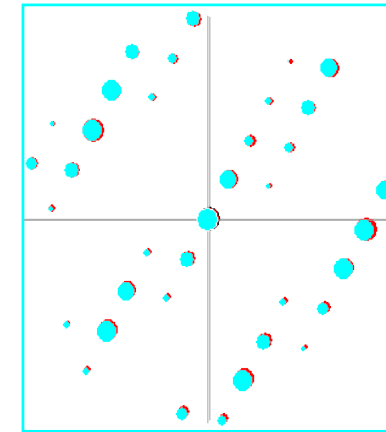
Orientation  $\Omega$



Orientation  $\Omega + \Omega' (= \Omega + 0.1^\circ)$




Bragg Spot pattern



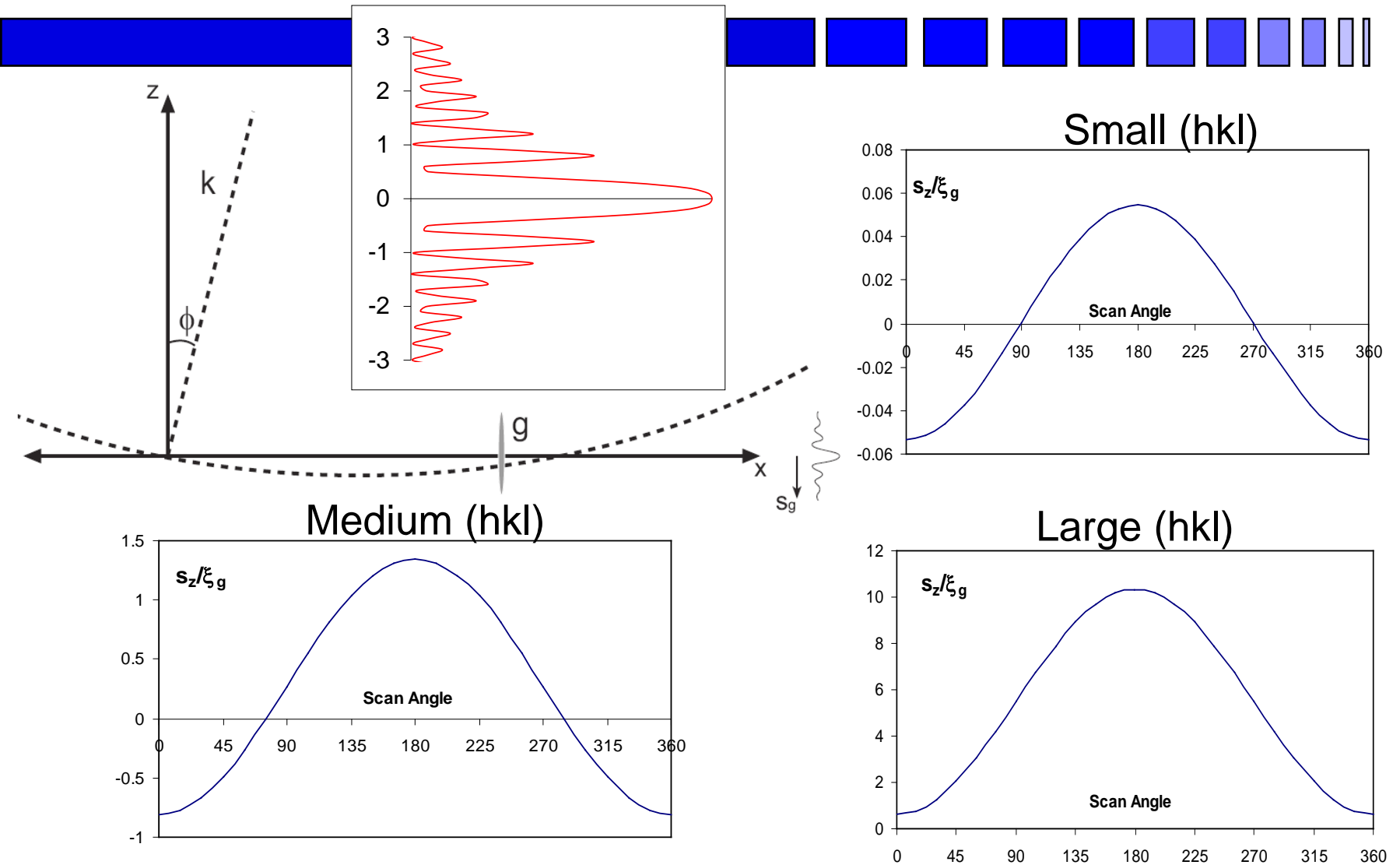
# Summary

- PED is
  - Approximately Invertable
  - Pseudo Bragg's Law
  - Approximately 2-beam, but not great
  - Properly Explained by Dynamical Theory
- PED is not
  - Pseudo-Kinematical (this is different!)
  - Fully Understood by Dummies, yet
- Important offshoots – ADT and OIM

# Speculation...Full Method

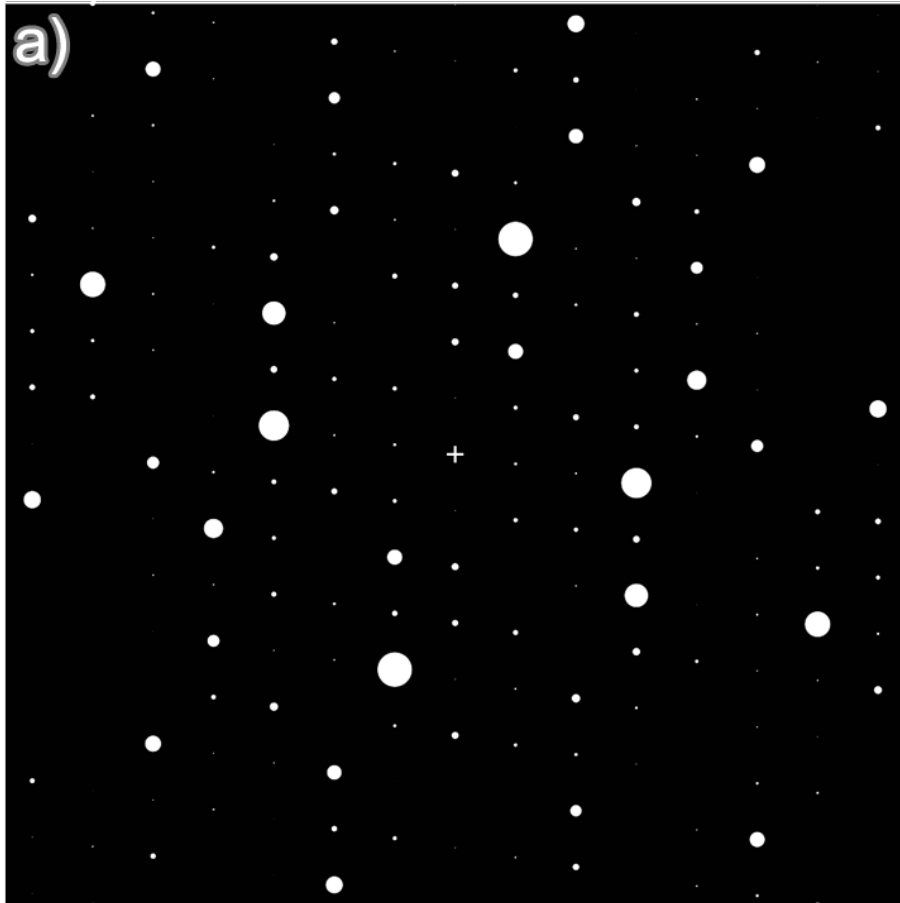
- 
- Generate initial structure estimate
    - Initial Bragg's Law Analysis or part of data

# Two-Beam Integration: Ewald Sphere

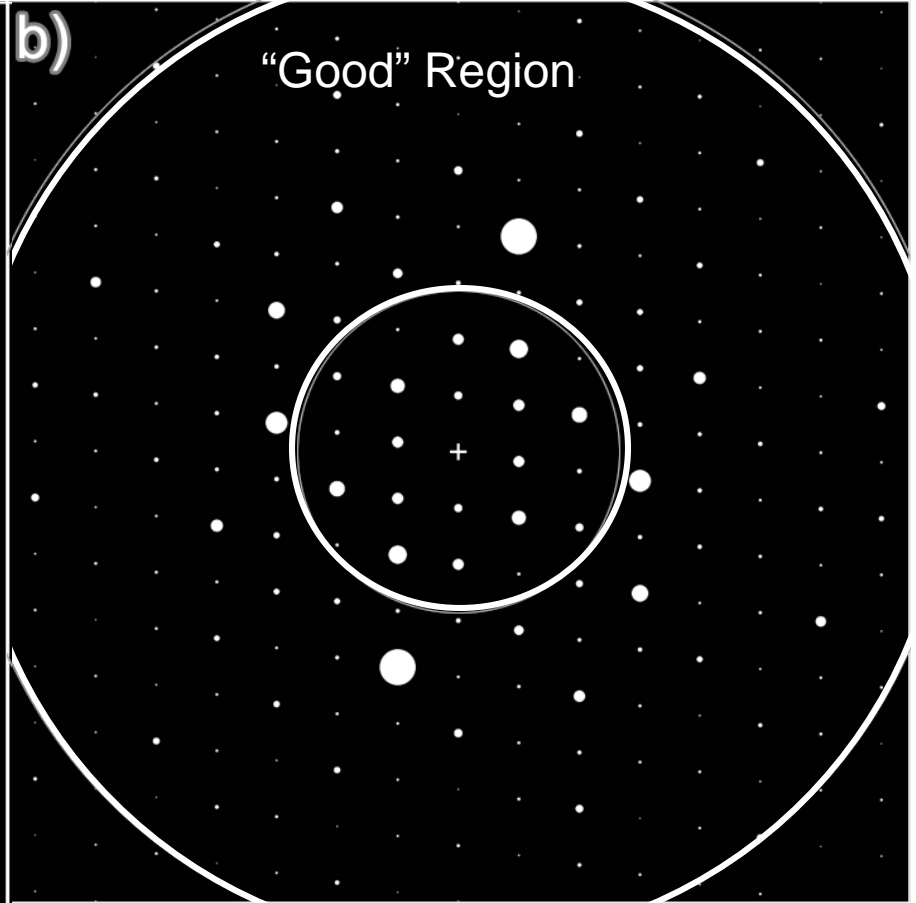




# Preprocess?




Bragg's Law (reference)

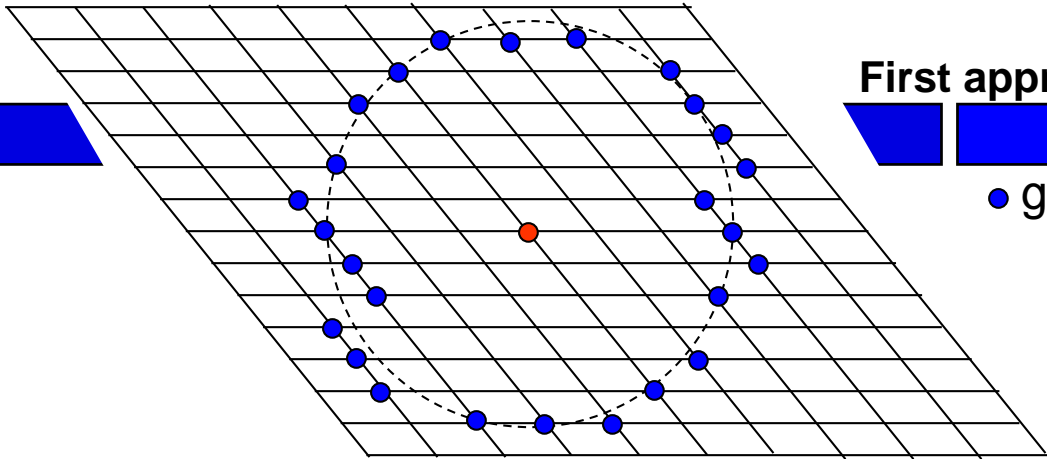


Precession pattern (experiment)  
 $\phi = 24\text{mrad}$

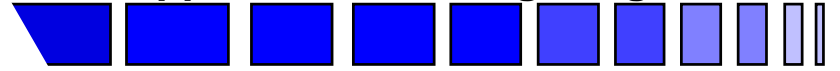
# Speculation...Full Method

- 
- Generate initial structure estimate
    - Initial Bragg's Law Analysis or part of data
    - Use 2-beam approximation to invert/correct
    - Partial refinement, using 1/2 kpoint Bloch-Wave method

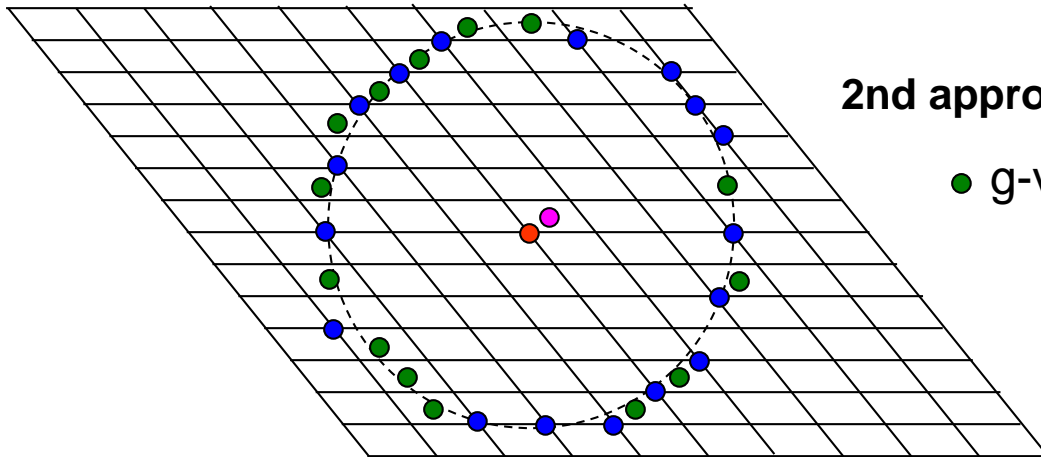
# Approximation of precession circuit by series of g-vector tilts:



First approximation; single eigen-solution



● g-vector tilts obtained from ●



2nd approximation; two eigen-solutions

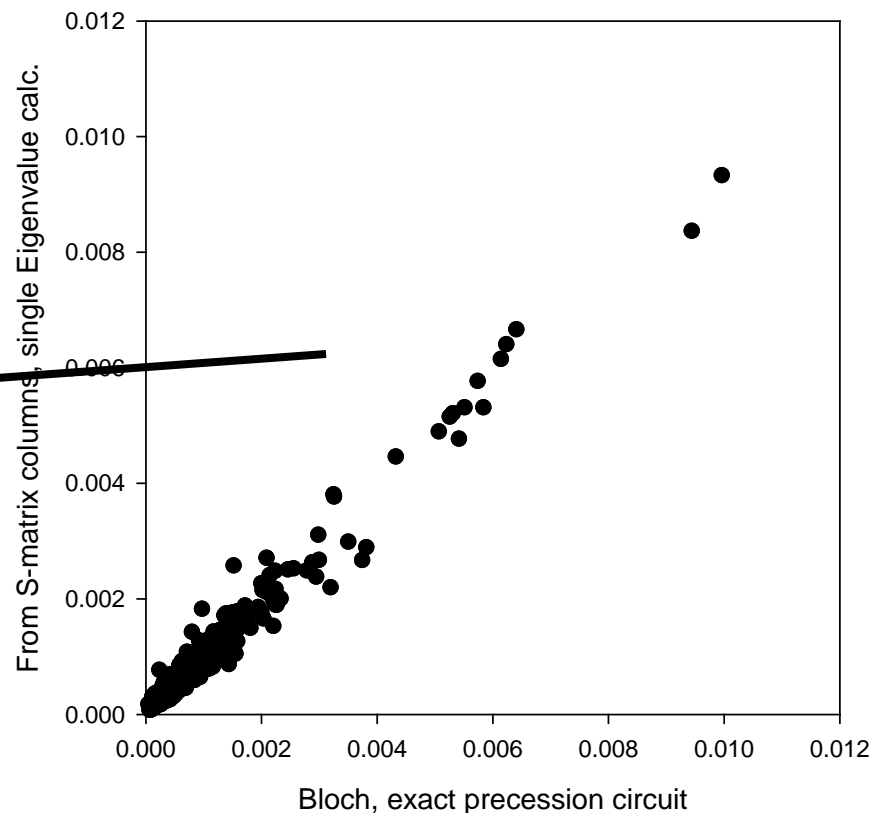
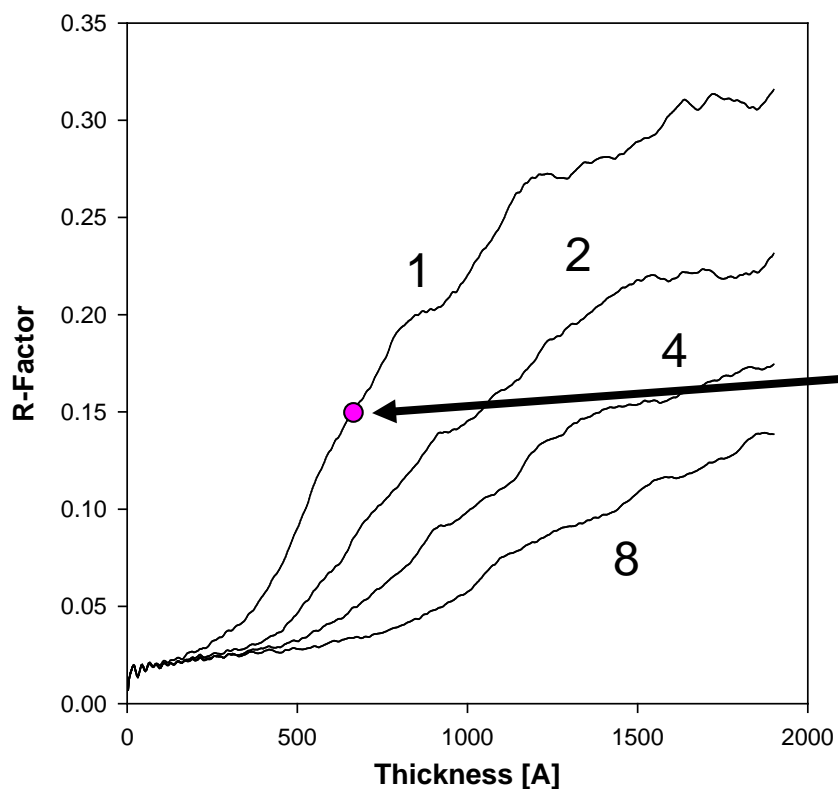
● g-vector tilts obtained from ●

=> increasingly accurate precession calculation


# Precession calculations for $(\text{Ga,In})_2\text{SnO}_5$ , 36 mrad semi-angle, increasing numbers of eigensolutions.



R-Factor agreement between exact precession circuit (1024 settings) and approximation using increasing numbers of Bloch calculations in 1st Brillouin Zone



# Speculation...Full Method

- 
- Generate initial structure estimate
    - Initial Bragg's Law Analysis or part of data
    - Use 2-beam approximation to invert/correct
    - Partial refinement, using 1/2 kpoint Bloch-Wave method
    - Final full refinement using 25-100 beam BW (+Bethe terms, e.g. Stadleman or NUMIS + dmnf, large residual code)

# Hypothesis: Thickness ranges



HREM



Bragg's Law



Dynamical

(not so easy)



PED (pseudo-Bragg's Law)



Questions ?